Lecturer cadre Syllabus (Maths)

Unit 1
Analysis

Set Theory:
Elementary set theory, finite, countable and uncountable sets, real number system as a complete ordered field, Archimedean property, supremum and infimum. Venn diagrams, union, intersection, difference and complement of sets, and properties, Bolzano–Weierstrass theorem.

Sequences & Series:
Sequences and series including Arithmetic Progression (A.P.), Arithmetic Mean (A.M.), Geometric Progression (G.P.), general term and sum of n terms of a G.P., arithmetic and geometric series, infinite geometric progression and its sum, Geometric Mean (G.M.), relation between A.M. and G.M., and special series ($\sum n$, $\sum n^2$, $\sum n^3$). Convergence of sequences and series, limit superior (limsup) and limit inferior (liminf), sequences and series of functions, uniform convergence.

Binomial theorem:
History ,Statement and proof for positive integral indices, Pascal's triangle, general and middle terms in binomial expansion, and simple applications.

Limits and derivatives:
Intuitive idea of limits, Continuity and uniform continuity, differentiability, chain rule, inverse trigonometric functions, implicit and parametric differentiation, exponential and logarithmic functions, logarithmic differentiation, second-order derivatives, and Rolle's and Lagrange's Mean Value Theorems.

Applications of derivatives:
Rate of change, increasing and decreasing functions, tangents and normals, approximation, maxima and minima, real-life problems.

Integration and Application of Integrals:
Integration as an inverse process of differentiation, methods: substitution, partial fractions, by parts, definite integrals as limits of sums, Fundamental Theorem of Calculus, properties and evaluation of definite integrals, area under curve, Riemann sums, Riemann integral, and improper integrals.

Advanced Analysis:
Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure and Lebesgue integration (basic concepts), metric spaces, compactness and connectedness, and Heine–Borel theorem, normed linear spaces, spaces of continuous functions.

Functions of several variables:
Partial derivatives, directional derivatives, Jacobians, derivative as a linear transformation, and inverse and implicit function theorems.

Linear Algebra

Vector spaces including vectors and scalars, magnitude and direction, direction ratios and cosines, types of vectors, position vector, vector operations including addition, scalar multiplication, dot product, **cross product and scalar triple product**.

Subspaces, linear dependence and independence, basis and dimension, linear transformations.

Matrices including types of matrices, operations, transpose, symmetric and skew-symmetric matrices, elementary operations, inverse of matrices. Determinants including **properties**, minors, cofactors and applications. System of linear equations, rank of matrix, consistency, eigenvalues and eigenvectors, Cayley–Hamilton theorem. Matrix representation of linear transformations, change of basis, canonical forms, diagonal and Jordan forms. Inner product spaces, orthonormal basis (Gram–Schmidt process), and quadratic forms.

Coordinate Geometry Straight lines including slope, angle between lines, various forms of equations of a line, distance of a point from a line, and family of lines. Conic sections including circle, parabola, ellipse and hyperbola, their standard equations and basic properties.

Three-Dimensional Geometry Coordinates in space, distance between two points, section formula. Direction ratios and cosines, equations of lines and planes, coplanar and skew lines, shortest distance between lines, angle between lines and planes, and distance of a point from a plane.

Linear Inequalities Linear inequalities in one and two variables, graphical solutions, and system of inequalities.

UNIT 2

Complex Analysis Complex numbers including the need for complex numbers ($\sqrt{-1}$) arising from the inability to solve all quadratic equations in the real number system, algebraic properties of complex numbers, and representation in the Argand plane. Polar form of complex numbers, modulus and argument, and square roots of complex numbers. Fundamental Theorem of Algebra and solutions of quadratic equations in the complex number system.

Polynomials and power series, and transcendental functions including exponential, trigonometric and hyperbolic functions, Analytic functions, Cauchy–Riemann equations. Complex integration, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum Modulus Principle, Schwarz lemma, Open mapping theorem. Singularities, residues, residue theorem. Taylor and Laurent series, analytic continuation, conformal mappings and Möbius transformations.

Algebra

Abstract Algebra:

Permutations and combinations including the fundamental principle of counting, factorial $n!$, permutations and combinations, derivation of formulae and their interrelations, and simple applications. Pigeon-hole principle and inclusion–exclusion principle.

Number theory including the fundamental theorem of arithmetic, divisibility in integers, congruences, Chinese Remainder Theorem, Euler's phi-function, and primitive roots.

Group theory:

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, and Sylow theorems.

Ring theory:
Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domains (UFD), principal ideal domains (PID), and Euclidean domains, Polynomial rings and irreducibility criteria, fields and finite fields, field extensions, and Galois theory.

Topology:
Topological spaces; basis; dense sets; subspace and product topology; separation axioms; connectedness and compactness.

Trigonometry

Trigonometric functions including positive and negative angles, measurement of angles in degrees and radians and conversion between them. Definition of trigonometric functions using the unit circle, signs of trigonometric functions, and graphs of trigonometric functions. Fundamental identities including $\sin^2 x + \cos^2 x = 1$.

Trigonometric identities including angle sum and difference formulas such as $\sin(x \pm y)$and $\cos(x \pm y)$, and identities related to multiple angles including $\sin 2x$, $\cos 2x$, $\tan 2x$, $\sin 3x$, $\cos 3x$and $\tan 3x$.

Trigonometric equations including general solutions of equations of the type $\sin\theta = \sin\alpha$, $\cos\theta = \cos\alpha$, and $\tan\theta = \tan\alpha$, and inverse trigonometric functions.

UNIT 3

Differential Equations

Ordinary differential equations:
Order and degree, general and particular solutions, and **formation of differential equations whose general solution is given**. Methods of solving first-order differential equations including separation of variables and homogeneous equations of first order and first degree. Linear differential equations of first order,

Existence and uniqueness of solutions of initial value problems, singular solutions, and systems of first-order differential equations.

Higher-order differential equations including general theory of homogeneous and non-homogeneous linear differential equations, method of variation of parameters, Sturm–Liouville boundary value problems, and Green's functions.

Partial differential equations:
Partial differential equations including first-order PDEs, Lagrange and Charpit methods, and Cauchy problem. Classification of second-order PDEs, general solutions of higher-order PDEs with constant coefficients, and method of separation of variables for solving Laplace, heat and wave equations.

Numerical Analysis

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Calculus of Variations & Integral Equations	Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations. Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.
Classical Mechanics	Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

UNIT 4

Descriptive Statistics & Data Analysis	Descriptive statistics and exploratory data analysis including measures of dispersion such as mean deviation, variance and standard deviation for grouped and ungrouped data, and analysis of frequency distributions with equal means but different variances.
Probability	Sample spaces and events, mutually exclusive and exhaustive events, axiomatic probability, probability of events, addition and multiplication theorems, conditional probability, independent events, total probability and Bayes' theorem. Random variables and their probability distributions, expectation, mean and variance, moments, repeated independent (Bernoulli) trials and binomial distribution. Joint distributions, marginal and conditional distributions, characteristic functions. Probability inequalities including Chebyshev, Markov and Jensen inequalities. Modes of convergence, weak and strong laws of large numbers, and Central Limit Theorem.
Stochastic Processes	Markov chains with finite and countable state space, classification of states, limiting behaviour of transition probabilities, stationary distributions, Poisson processes and birth–death processes.
Statistical Inference	Standard discrete and continuous distributions, sampling distributions, standard errors, asymptotic distributions, order statistics and range. Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses including most powerful and uniformly most powerful tests, likelihood ratio tests, chi-square tests of goodness of fit, large sample tests, non-parametric tests, rank correlation, test for independence, and elementary Bayesian inference.
Linear Models & Multivariate Analysis	Gauss–Markov theorem, estimability of parameters, best linear unbiased estimators (BLUE), confidence intervals and tests for linear hypotheses. Analysis of variance (ANOVA) and covariance (ANCOVA), fixed, random and mixed effects models. Simple and multiple linear regression, regression diagnostics, logistic regression. Multivariate normal distribution, Wishart distribution, quadratic forms, inference for parameters, partial and multiple correlation coefficients. Data reduction, techniques including principal component analysis, discriminant analysis, cluster analysis and canonical correlation.
Sampling Techniques	Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling, ratio and regression methods.
Design of Experiments	Completely randomized design, randomized block design, Latin square design, connectedness and orthogonality of block designs, balanced incomplete block design (BIBD), factorial experiments including confounding and construction.

Operations Research	Linear programming problems including formulation, constraints, objective functions, graphical solutions, feasible and optimal solutions. Simplex method and duality. Transportation and assignment problems. Queuing models and inventory models, including steady-state solutions of Markovian queuing systems such as M/M/1, M/M/C and M/G/1.
Mathematical Reasoning	Mathematically acceptable statements, logical connectives including "if and only if", "implies", "and", "or", "there exists", and their use in mathematical statements. Validation of statements including contradiction, converse and contrapositive.