

Arithmetic Progression

➤ **Series of the form**→

$a, a + d, a + 2d, a + 3d, \dots$

is called Arithmetic Progression, when they increase or decrease by a common difference

e.g.→ 5, 12, 19, 26, 33

Common difference, $d = 12 - 5 = 7$

$$19 - 12 = 7$$

$$26 - 19 = 7$$

➤ $a_1, a_2, a_3, \dots, T_n$

$d =$ common difference

$=$ 2nd term - 1st term

$$= a_2 - a_1$$

$$T_n = a + (n - 1)d$$

$a \rightarrow$ 1st term

$n \rightarrow$ no. of terms

$d \rightarrow$ Common difference

➤ Sum of n terms in A.P

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Or

$$S_n = \frac{n}{2}[\text{First term} + \text{last term}]$$

➤ Total no. of term

$$= \frac{\text{Last term} - \text{First term}}{\text{Common difference}} + 1$$

Q1. 1, 4, 6, 5, 11, 6, Find the sum first 100 terms?

1, 4, 6, 5, 11, 6, पहले 100 पदों का योग ज्ञात कीजिए?

- (a) 7600
- (b) 7800
- (c) 7900
- (d) 8000

S1. Ans.(a)

Sol.

$$\begin{aligned}
&1 + 4 + 6 + 5 + 11 + 6 + \dots \dots \dots 100 \text{ terms} \\
&= [1 + 6 + 11 + \dots \dots \dots 50 \text{ terms}] + [4 + 5 + 6 + \dots \dots \dots + 50 \text{ terms}] \\
&= 502[2 \times 1 + (50 - 1) 5] + 502[2 \times 4 + (50 - 1) \times 1] \\
&= 6175 + 1425 \\
&= 7600
\end{aligned}$$

Q2. Find the sum of all 2 digits no. which will exactly divided by 9?

सभी 2 अंकों की संख्या का योग ज्ञात कीजिए जो ठीक 9 से विभाज्य होंगे?

- (a) 565
- (b) 585
- (c) 525
- (d) 575

S2. Ans.(b)

Sol.

Number exactly divisible by 9

$$= 18 + 27 + 36 + 45 + \dots \dots \dots 99$$

No. of terms, $n = 99 - 18 + 1 = 81 + 1 = 82$

Sum of the series = $n/2 [1st \text{ term} + \text{last term}] = 82/2 [18 + 99]$

$$= 585$$

Q3. If $T_2 + T_5 = 8$ of an A.P & $T_3 + T_7 = 14$ of that A.P then, find the 11th term?

यदि एक अंकगणितीय प्रगति का $T_2 + T_5 = 8$ होता है और उस अंकगणितीय प्रगति का $T_3 + T_7 = 14$ होता है तो उसका 11वां पद ज्ञात कीजिए?

- (a) 20
- (b) 21
- (c) 22
- (d) 19

S3. Ans.(d)

Sol.

$$\begin{aligned}
T_2 + T_5 &= 8 \\
a + d + a + 4d &= 8 \\
2a + 5d &= 8 \dots(i) \\
T_3 + T_7 &= 14 \\
a + 2d + a + 6d &= 14 \\
2a + 8d &= 14 \dots(ii) \\
\text{From (i) \& (ii)} \\
3d &= 6 \\
d &= 2 \\
2a + 10 &= 8 \\
2a &= -2 \\
a &= -1 \\
T_{11} &= a + 10d \\
&= -1 + 20 = 19
\end{aligned}$$

Q4. If $t_1 + t_5 + t_{10} + t_{20} + t_{24} = 225$, Find the sum of first 23rd term of that A.P?

यदि $t_1 + t_5 + t_{10} + t_{20} + t_{24} = 225$ तो उस अंकगणितीय प्रगति के पहले 24 वें पदों का योग ज्ञात कीजिए?

- (a) 800
- (b) 700
- (c) 850
- (d) 1035

S4. Ans.(d)

Sol.

$$a + a + 4d + a + 9d + a + 19d + a + 23d = 225$$

$$5a + 55d = 225$$

$$a + 11d = 45$$

$$S_{23} = 23/2 [2a + 22d]$$

$$= 23/2 \times 90 = 1035$$

Geometric Progression

➤ $a, ar, ar^2, ar^3, \dots, ar^{n-1}$

Common Ratio (r) = $\frac{\text{Second term}}{\text{First term}}$

n^{th} term (T_n) = ar^{n-1}

➤ Sum of G.P, $S_n = \frac{a(r^n-1)}{r-1}$ where $r > 1$

Or

$S_n = \frac{a(1-r^n)}{1-r}$ where $r < 1$

➤ Sum of an Infinite Geometric Progression when $r < 1$

$$S_\infty = \frac{a}{1-r}$$

Q1. The 7th term of G.P is 8 times the 4th term. What will be the 1st term if its 5th term is 48?

ज्यामितीय प्रगति का सातवाँ पद चौथे पद का 8 गुना है. पहला पद क्या होगा यदि उसका पांचवाँ पद 48 है?

- (a) 5
- (b) 6
- (c) 3
- (d) 7

S1. Ans.(c)

Sol.

$$T_7 = 8 \times T_4$$

$$ar^6 = 8 \times ar^3$$

$$r^3 = 8$$

$$r = 2$$

$$T_5 = 48$$

$$ar^4 = 48$$

$$a \times 16 = 48$$

$$a = 3$$

Q2. Find the value of $25^{\left[\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \dots \dots \infty\right]}$?

$25^{\left[\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \dots \dots \infty\right]}$ का मान ज्ञात कीजिए?

- (a) 5
- (b) 6
- (c) 7
- (d) 9

S2. Ans.(a)

Sol.

$25^{\left[\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \dots \dots \infty\right]}$

$$S_{\infty} = a/1-r = \frac{1/3}{1-1/3} = 1/2 \Rightarrow 25^{1/2} = 5$$

Some Important Formulas: -

➤ To find the sum of First n natural number.

$$S = 1 + 2 + 3 + 4 + \dots \dots \dots n$$

$$S = \frac{n(n+1)}{2}$$

➤ To find the sum of the Squares of the 1st n natural numbers.

$$S = 1^2 + 2^2 + 3^2 + \dots \dots \dots + n^2$$

$$S = \frac{n(n+1)(2n+1)}{6}$$

➤ To find the sum of the cubes of the 1st n natural numbers.

$$S = 1^3 + 2^3 + 3^3 + \dots \dots \dots + n^3$$

$$S = \left[\frac{n(n+1)}{2}\right]^2$$

➤ To find the sum of first n odd natural numbers.

$$S = 1 + 3 + 5 + \dots \dots \dots + (2n - 1)$$

$$S = n^2$$

➤ To find the sum of first n even natural numbers

$$S = 2 + 4 + 6 + \dots \dots \dots + 2n$$

$$S = n(n+1)$$

Q1. The sum of first 29 odd natural number is equal to?

पहली 29 विषम संख्याओं का योग बराबर है:

- (a) 1000
- (b) 625

- (c) 729
(d) 841

S1. Ans.(d)

Sol. Sum of 1st 27 odd natural

$$\begin{aligned}\text{Number} &= (n)^2 \\ &= 841\end{aligned}$$

Q2. The Sum of first 100 even natural numbers is equal to?

पहली 100 सम संख्याओं का योग बराबर है:

- (a) 10100
(b) 10200
(c) 10300
(d) 10400

S2. Ans.(a)

Sol. Sum of 1st 100 even natural

$$\begin{aligned}\text{Number} &= (n) (n + 1) \\ &= 100 \times 101 \\ &= 10100\end{aligned}$$

Q3. The sum of squares of first 50 natural numbers is equal to?

पहली 50 प्राकृतिक संख्याओं के वर्गों का योग बराबर है:

- (a) 42925
(b) 43935
(c) 44945
(d) 45955

S3. Ans.(a)

Sol. Sum of the squares of 1st 50 natural

$$\begin{aligned}\text{Number} &= \frac{(n)(n+1)(2n+1)}{6} \\ &= \frac{50 \times 51 \times 101}{6} \\ &= 42925\end{aligned}$$

Q4. The sum of first 500 natural numbers is equal to?

पहली 500 प्राकृतिक संख्याओं का योग बराबर है:

- (a) 125250
(b) 124240
(c) 126260
(d) 127270

S4. Ans.(a)

Sol. Sum of the 1st 800 natural Numbers

$$\begin{aligned} &= \frac{n(n+1)}{2} \\ &= \frac{500 \times 501}{2} = 125250 \end{aligned}$$

Q5. The sum of cubes of first 30 numbers is equal to?

पहली 500 प्राकृतिक संख्याओं का योग बराबर है:

- (a) 210225
- (b) 216225
- (c) 400225
- (d) 420225

S5. Ans.(b)

Sol. Sum of cubes of 1st 30 numbers

$$= \left[\frac{n(n+1)}{2} \right]^2 = \left[\frac{30 \times 31}{2} \right]^2 = 216225$$

