Series : SSJ/2

SET – 4

प्रश्न पत्र कोड नं.
Question Paper Code No. **065/2/4**

रोल नं.
Roll No.

1	7	8	3	5	3	6	4

परीक्षार्थी QP कोड को OMR उत्तर-पत्रक के मुख-पृष्ठ पर अवश्य लिखें/भरें।
Candidates must write / fill the QP Code in the space allotted on OMR Sheet.

नोट / NOTE :

(i) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 24 हैं।
Please check that this question paper contains 24 printed pages.

(ii) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 50 बहुविकल्पीय प्रश्न (MCQs) हैं।
Please check that this question paper contains 50 Multiple Choice Questions (MCQs.)

(iii) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए QP कोड नम्बर को छात्र OMR शीट में उपयुक्त स्थान पर लिखें।
QP Code given on the right hand side of the question paper should be written on the appropriate place of the OMR Sheet by the candidates.

(iv) परीक्षा शुरू होने के वास्तविक समय से पहले इस प्रश्न-पत्र को पढ़ने के लिए 20 मिनट का अतिरिक्त समय आबंटित किया गया है।
20 minutes additional time has been allotted to read this question paper prior to actual time of commencement of examination.

गणित
MATHEMATICS
सत्र – I / Term – I

निर्धारित समय : 90 मिनट
Time allowed : 90 Minutes

अधिकतम अंक : 40
Maximum Marks : 40

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ENGLISH VERSION

General Instructions :

(i) *This question paper comprises **50** questions out of which **40** questions are to be attempted as per instructions. All questions carry equal marks.*

(ii) *The question paper consists of three Sections – Section A, B and C.*

(iii) ***Section – A** contains **20** questions. Attempt any **16** questions from Q. No. **1** to **20**.*

(iv) ***Section – B** also contains **20** questions. Attempt any **16** questions from Q. No. **21** to **40**.*

(v) ***Section – C** contains **10** questions including one Case Study. Attempt any **8** from Q. No. **41** to **50**.*

(vi) *There is only one correct option for every Multiple Choice Question (MCQ). Marks will not be awarded for answering more than one option.*

(vii) *There is no negative marking.*

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SECTION – A

In this section, attempt any **16** questions out of Questions **1-20**. Each question is of **one** mark.

1. Differential of log [log (log x^5)] w.r.t. x is

(a) $\dfrac{5}{x \log (x^5) \log (\log x^5)}$ (b) $\dfrac{5}{x \log (\log x^5)}$

(c) $\dfrac{5x^4}{\log (x^5) \log (\log x^5)}$ (d) $\dfrac{5x^4}{\log x^5 \log (\log x^5)}$

2. The number of all possible matrices of order 2×3 with each entry 1 or 2 is

(a) 16 (b) 6

(c) 64 (d) 24

3. A function $f : R \rightarrow R$ is defined as $f(x) = x^3 + 1$. Then the function has

(a) no minimum value

(b) no maximum value

(c) both maximum and minimum values

(d) neither maximum value nor minimum value

4. If $\sin y = x \cos (a + y)$, then $\dfrac{dx}{dy}$ is

(a) $\dfrac{\cos a}{\cos^2 (a + y)}$ (b) $\dfrac{-\cos a}{\cos^2 (a + y)}$

(c) $\dfrac{\cos a}{\sin^2 y}$ (d) $\dfrac{-\cos a}{\sin^2 y}$

5. The points on the curve $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$, where tangent is parallel to x-axis are

(a) $(\pm 5, 0)$ (b) $(0, \pm 5)$

(c) $(0, \pm 3)$ (d) $(\pm 3, 0)$

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6. Three points P(2x, x + 3), Q(0, x) and R(x + 3, x + 6) are collinear, then x is equal to

(a) 0

(b) 2

(c) 3

(d) 1

7. The principal value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{12}$

(b) π

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{6}$

8. If $(x^2 + y^2)^2 = xy$, then $\frac{dy}{dx}$ is

(a) $\frac{y + 4x(x^2 + y^2)}{4y(x^2 + y^2) - x}$

(b) $\frac{y - 4x(x^2 + y^2)}{x + 4(x^2 + y^2)}$

(c) $\frac{y - 4x(x^2 + y^2)}{4y(x^2 + y^2) - x}$

(d) $\frac{4y(x^2 + y^2) - x}{y - 4x(x^2 + y^2)}$

9. If a matrix A is both symmetric and skew symmetric, then A is necessarily a

(a) Diagonal matrix

(b) Zero square matrix

(c) Square matrix

(d) Identity matrix

10. Let set X = {1, 2, 3} and a relation R is defined in X as : R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are

(a) {(1, 1), (2, 3), (1, 2)}

(b) {(3, 3), (3, 1), (1, 2)}

(c) {(1, 1), (3, 3), (3, 1), (2, 3)}

(d) {(1, 1), (3, 3), (3, 1), (1, 2)}

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11. A Linear Programming Problem is as follows :

Minimise $z = 2x + y$

subject to the constraints $x \geq 3,\ x \leq 9,\ y \geq 0$

$x - y \geq 0,\ x + y \leq 14$

The feasible region has

(a) 5 corner points including (0, 0) and (9, 5)

(b) 5 corner points including (7, 7) and (3, 3)

(c) 5 corner points including (14, 0) and (9, 0)

(d) 5 corner points including (3, 6) and (9, 5)

12. The function $f(x) = \begin{cases} \dfrac{e^{3x} - e^{-5x}}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$

is continuous at $x = 0$ for the value of k, as

(a) 3 (b) 5

(c) 2 (d) 8

13. If C_{ij} denotes the cofactor of element p_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$,

then the value of $C_{31} \cdot C_{23}$ is

(a) 5 (b) 24

(c) -24 (d) -5

14. The function $y = x^2 e^{-x}$ is decreasing in the interval

(a) $(0, 2)$ (b) $(2, \infty)$

(c) $(-\infty, 0)$ (d) $(-\infty, 0) \cup (2, \infty)$

15. If $R = \{(x, y);\ x, y \in Z,\ x^2 + y^2 \leq 4\}$ is a relation in set Z, then domain of R is

(a) $\{0, 1, 2\}$ (b) $\{-2, -1, 0, 1, 2\}$

(c) $\{0, -1, -2\}$ (d) $\{-1, 0, 1\}$

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16. The system of linear equations

$$5x + ky = 5,$$
$$3x + 3y = 5;$$

will be consistent if

(a) $k \neq -3$ (b) $k = -5$

(c) $k = 5$ (d) $k \neq 5$

17. The equation of the tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses the x-axis is

(a) $x - 5y = 2$ (b) $5x - y = 2$

(c) $x + 5y = 2$ (d) $5x + y = 2$

18. If $\begin{bmatrix} 3c + 6 & a - d \\ a + d & 2 - 3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is

(a) 4 (b) 16

(c) -4 (d) -16

$c = 2$

$a - d = 2$ $-3 - d = 2$

$a + d = -8$ $-d = 5$

$2a = -6$ $d = -5$

$a = -3$

19. The principal value of $\tan^{-1}\left(\tan \frac{9\pi}{8}\right)$ is

(a) $\frac{\pi}{8}$ (b) $\frac{3\pi}{8}$

(c) $-\frac{\pi}{8}$ (d) $-\frac{3\pi}{8}$

$2 - 3b = -4$

$-3b = -6$

$b = 3$

20. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ $P - Q$ is

2×22.23

$-9 + 10$

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

$\begin{matrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{matrix}$ $\begin{matrix} -1 & 1 \\ 2 \times 2 & 2 \\ 3 \times 1 & 3 \end{matrix}$

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SECTION – B

In this Section attempt any **16** questions out of the Questions **21-40**. Each question is of **one** mark.

21. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval

(a) $(-\infty, 2) \cup (3, \infty)$
(b) $(-\infty, 2)$
(c) $(-\infty, 2] \cup [3, \infty)$
(d) $[3, \infty)$

22. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then $\frac{dy}{dx}$ is

(a) $\frac{\cos\theta + \cos 2\theta}{\sin\theta - \sin 2\theta}$
(b) $\frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$
(c) $\frac{\cos\theta - \cos 2\theta}{\sin\theta - \sin 2\theta}$
(d) $\frac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta}$

23. What is the domain of the function $\cos^{-1}(2x - 3)$?

(a) $[-1, 1]$
(b) $(1, 2)$
(c) $(-1, 1)$
(d) $[1, 2]$

24. A matrix $A = [a_{ij}]_{3 \times 3}$ is defined by

$$a_{ij} = \begin{cases} 2i + 3j & , i < j \\ 5 & , i = j \\ 3i - 2j & , i > j \end{cases}$$

The number of elements in A which are more than 5, is

(a) 3
(b) 4
(c) 5
(d) 6

25. If a function f defined by

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3 & , \text{if } x = \frac{\pi}{2} \end{cases}$$

$\frac{k\cos x}{\pi - 2x} = 3$

$\frac{-k\sin x}{-2} = 3$

$\frac{-k}{-2} = 3$, $-k = -6$

$k = 6$

is continuous at $x = \frac{\pi}{2}$, then the value of k is

(a) 2
(b) 3
(c) 6
(d) -6

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26. For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $(X^2 - X)$ is

(a) 2I (b) 3I
(c) I (d) 5I

27. Let $X = \{x^2 : x \in N\}$ and the function $f : N \to X$ is defined by $f(x) = x^2, x \in N$. Then this function is

(a) injective only (b) not bijective
(c) surjective only (d) bijective

28. The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0). The minimum value of the objective function $Z = 2x + 5y$ is at the point

(a) P (b) Q
(c) R (d) S

29. The equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3) is

(a) $2y - 3mx + am^3 = 0$ (b) $2x + 3my - 3am^4 - am^2 = 0$
(c) $2x + 3my + 3am^4 - 2am^2 = 0$ (d) $2x + 3my - 3am^4 - 2am^2 = 0$

30. If A is a square matrix of order 3 and $|A| = -5$, then $|\text{adj } A|$ is

(a) 125 (b) -25
(c) 25 (d) ± 25

$2x + 5y$

31. The simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ is

(a) $\frac{\pi}{4} - \frac{x}{2}$ (b) $\frac{\pi}{4} + \frac{x}{2}$
(c) $\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$ (d) $\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x$

32. If for the matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is

(a) ± 3 (b) -3
(c) ± 1 (d) 1

$|A^3| = 125$

$|A| = 5$

$\begin{bmatrix} 0 & -2 \\ -2 & \alpha \end{bmatrix}$

$\begin{bmatrix} 1 & 6 \\ 6 & 4 \end{bmatrix}$

$= 16 - 2(6) = -2$

$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} =$

$a^2 - 4 = 5$

$a^2 = 9 \quad a = \pm 3$

$4 - 6 = -8$

$\left[\frac{\cos x/2 - \sin x/2}{\cos x/2 + \sin x/2}\right.$

$= \frac{1 - \tan x}{1 + \tan x}$

$\frac{1 - \tan a}{1 + \tan a \tan b}$

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33. If $y = \sin(m \sin^{-1} x)$, then which one of the following equations is true ?

(a) $(1 - x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + m^2y = 0$

(b) $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

(c) $(1 + x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$

(d) $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2x = 0$

34. The principal value of $[\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})]$ is

(a) π

(b) $-\frac{\pi}{2}$

(c) 0

(d) $2\sqrt{3}$

$\frac{\pi}{3} - (\pi - \frac{\pi}{6})$

$= \frac{\pi}{3} - \frac{5\pi}{6}$

35. The maximum value of $\left(\frac{1}{x}\right)^x$ is

(a) $e^{1/e}$

(b) e

(c) $\left(\frac{1}{e}\right)^{1/e}$

(d) e^e

36. Let matrix $X = [x_{ij}]$ is given by $X = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$. Then the matrix $Y = [m_{ij}]$, where m_{ij} = Minor of x_{ij}, is

-19

12

(a) $\begin{bmatrix} 7 & -5 & -3 \\ 19 & 1 & -11 \\ -11 & 1 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & -19 & -11 \\ 5 & -1 & -1 \\ 3 & 11 & 7 \end{bmatrix}$

$3 + 2 = 5$

$-1 + 2 = -1$

(c) $\begin{bmatrix} 7 & 19 & -11 \\ -3 & 11 & 7 \\ -5 & -1 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

$-3 \longrightarrow 11$

$-2 \longrightarrow 6$

$-1 \longrightarrow 3$

$0 \longrightarrow 0$

1

2

3

37. A function $f : R \to R$ defined by $f(x) = 2 + x^2$ is

(a) not one-one

(b) one-one

(c) not onto

(d) neither one-one nor onto

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38. A Linear Programming Problem is as follows :

Maximise / Minimise objective function $Z = 2x - y + 5$

Subject to the constraints

$$3x + 4y \le 60$$
$$x + 3y \le 30$$
$$x \ge 0,\ y \ge 0$$

If the corner points of the feasible region are A (0, 10), B(12, 6), C(20, 0) and O(0, 0), then which of the following is true ?

(a) Maximum value of Z is 40

(b) Minimum value of Z is – 5

(c) Difference of maximum and minimum values of Z is 35

(d) At two corner points, value of Z are equal

39. If $x = -4$ is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is

(a) 4 (b) – 3

(c) 2 (d) 5

40. The absolute maximum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in the interval $\left[-2, \frac{9}{2}\right]$ is

(a) 8 (b) 9

(c) 6 (d) 10

SECTION – C

Attempt any 8 questions out of the Questions **41-50**. Each question is of **one mark.**

41. In a sphere of radius r, a right circular cone of height h having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is

(a) $2\pi^2 rh\,(2rh + h^2)$ (b) $\pi^2 hr\,(2rh + h^2)$

(c) $2\pi^2 r\,(2rh^2 - h^3)$ (d) $2\pi^2 r^2\,(2rh - h^2)$

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42. The corner points of the feasible region determined by a set of constraints (linear inequalities) are P(0, 5), Q(3, 5), R(5, 0) and S(4, 1) and the objective function is $Z = ax + 2by$ where $a, b > 0$. The condition on a and b such that the maximum Z occurs at Q and S is

(a) $a - 5b = 0$
(b) $a - 3b = 0$
(c) $a - 2b = 0$
(d) $a - 8b = 0$

43. If curves $y^2 = 4x$ and $xy = c$ cut at right angles, then the value of c is

(a) $4\sqrt{2}$
(b) 8
(c) $2\sqrt{2}$
(d) $-4\sqrt{2}$

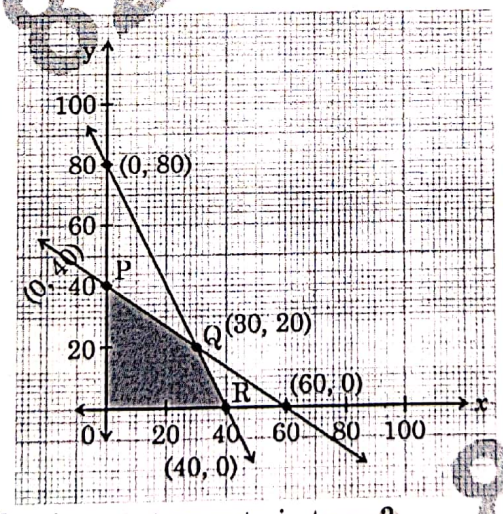
$y^2 = 4x$
$2y \cdot = 4$
$y' = \frac{2}{y}$

44. The inverse of the matrix $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(a) $24\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$
(b) $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(c) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$
(d) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

$xy = c$
$xy' + y' = c$
$y'(x+1) = c$
$y' = \frac{c}{x+1}$

45. For an L.P.P. the objective function is $Z = 4x + 3y$, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



$80 +$
$120 + 60$

Which one of the following statements is true ?

(a) Maximum value of Z is at R.
(b) Maximum value of Z is at Q.
(c) Value of Z at R is less than the value at P.
(d) Value of Z at Q is less than the value at R.

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Case Study

Compost Pit

In a residential society comprising of 100 houses, there were 60 children between the ages of 10-15 years. They were inspired by their teachers to start composting to ensure that biodegradable waste is recycled. For this purpose, instead of each child doing it for only his/her house, children convinced the Residents welfare association to do it as a society initiative. For this they identified a square area in the local park. Local authorities charged amount of ₹ 50 per square metre for space so that there is no misuse of the space and Resident welfare association takes it seriously. Association hired a labourer for digging out 250 m^3 and he charged ₹ 400 × $(\text{depth})^2$. Association will like to have minimum cost.

Based on this information, answer the any 4 of the following questions.

46. Let side of square plot is x m and its depth is h metres, then cost c for the pit is

(a) $\frac{50}{h} + 400\,h^2$

(b) $\frac{12500}{h} + 400\,h^2$

(c) $\frac{250}{h} + h^2$

(d) $\frac{250}{h} + 400\,h^2$

47. Value of h (in m) for which $\frac{dc}{dh} = 0$ is

400×d²

(a) 1.5

(b) 2

(c) 2.5

(d) 3

48. $\frac{d^2c}{dh^2}$ is given by

(a) $\frac{25000}{h^3} + 800$

(b) $\frac{500}{h^3} + 800$

(c) $\frac{100}{h^3} + 800$

(d) $\frac{500}{h^3} + 2$

49. Value of x (in m) for minimum cost is

(a) 5

(b) $10\sqrt{\frac{5}{3}}$

(c) $5\sqrt{5}$

(d) 10

50. Total minimum cost of digging the pit (in ₹) is

(a) 4,100

(b) 7,500

(c) 7,850

(d) 3,220

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