Solutions

S1. Ans.(a)

Sol.

$$2x + \frac{1}{4x} = 1$$

Dividing by 2 both side

$$x + \frac{1}{8x} = \frac{1}{2}$$

Squaring both side

$$x^2 + \frac{1}{64x^2} + 2 \times x \times \frac{1}{8x} = \frac{1}{4}$$

$$x^2 + \frac{1}{64x^2} = \frac{1}{4} - \frac{1}{4} = 0$$

S2. Ans.(b)

Sol.

$$\sqrt{x} - \sqrt{y} = 1 \qquad \text{... (i)}$$

$$\sqrt{x} + \sqrt{y} = 17 \qquad \text{... (ii)}$$

From equation (i) & equation (ii)

$$\sqrt{x} = 9, \sqrt{y} = 8$$

So, $\sqrt{xy} = 9 \times 8 = 72$

S3. Ans.(c)

Sol.

$$x = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$\text{So,} \left(x - \frac{\sqrt{126}}{\sqrt{42}}\right)\left(x - \frac{1}{x - \frac{2\sqrt{3}}{3}}\right)$$

$$= \left(\frac{4}{\sqrt{3}} - \frac{\sqrt{126}}{\sqrt{42}}\right)\left(x - \frac{1}{\frac{4}{\sqrt{3}} - \frac{2}{\sqrt{3}}}\right)$$

$$= \left(\frac{4\sqrt{42} - \sqrt{126} \times \sqrt{3}}{\sqrt{3} \times \sqrt{42}}\right)\left(x - \frac{1}{2/\sqrt{3}}\right)$$

$$= \frac{4\sqrt{42} - 3\sqrt{42}}{\sqrt{3} \times \sqrt{42}}\left(x - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{4\sqrt{42} - 3\sqrt{42}}{\sqrt{3} \times \sqrt{42}}\left(\frac{4}{\sqrt{3}} - \frac{\sqrt{3}}{2}\right)$$

$$= \left(\frac{\sqrt{42}}{\sqrt{3} \times \sqrt{42}}\right)\left(\frac{8-3}{2\sqrt{3}}\right) = \frac{5}{6}$$

S4. Ans.(b)

Sol.

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$56 = 2(a^2 + b^2 + ab)$

$28 = a^2 + b^2 + ab$ ... (i)

$a - b = 2$

$a^2 + b^2 - 2ab = 4$ ... (ii)

From equation (i) and equation (ii)

$3ab = 24$

$ab = 8$ ... (iii)

From equation (iii) and equation (i)

$a^2 + b^2 = 20$

S5. Ans.(c)

Sol. $x^2 - 3x + 1 = 0$

Divide by x

$$x - 3 + \frac{1}{x} = 0$$

$$x + \frac{1}{x} = 3$$

S6. Ans.(d)

Sol.

$$\frac{2+a}{a} + \frac{2+b}{b} + \frac{2+c}{c} = 4$$

$$\frac{2bc + abc + 2ac + abc + 2ab + abc}{abc} = 4$$

$$\frac{2(bc + ab + ca)}{abc} + \frac{3abc}{abc} = 4$$

$$\frac{2(bc + ab + ca)}{abc} + 3 = 4, \frac{ab + bc + ca}{abc} = \frac{1}{2}$$

S7. Ans.(b)

Sol.

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

$$\frac{xy + yz + zx}{xyz} = 1$$

$xy + yz + zx = xyz = -1$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$(1)^2 = x^2 + y^2 + z^2 + 2(-1)$

$x^2 + y^2 + z^2 = 3$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)[x^2 + y^2 + z^2 - (xy + yx + zx)]$

$x^3 + y^3 + z^3 - 3(-1) = 1[3 - (-1)]$

$x^3 + y^3 + z^3 = 4 - 3 = 1$

S8. Ans.(c)

Sol.

$x + \frac{1}{x} = 5$ then $\frac{5x}{x^2 + 5x + 1}$

$= \frac{5}{x + 5 + \frac{1}{x}} = \frac{5}{x + \frac{1}{x} + 5}$

$= \frac{5}{5 + 5} = \frac{1}{2}$

S9. Ans.(b)

Sol.

$\frac{1}{a}(a^2 + 1) = 3$

$a + \frac{1}{a} = 3$

$a^3 + \frac{1}{a^3} = (3)^3 - 3 \times 3$

$= 27 - 9 = 18$

$\frac{a^6 + 1}{a^3} = 18$

S10. Ans.(b)

Sol.

$a = \sqrt{2} + 1, b = \sqrt{2} - 1$

$\frac{1}{a+1} + \frac{1}{b+1} = \frac{1}{\sqrt{2} + 1 + 1} + \frac{1}{\sqrt{2} - 1 + 1} = \frac{1}{\sqrt{2} + 2} + \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}(\sqrt{2} + 1)} + \frac{1}{\sqrt{2}}$

$= \frac{1 + \sqrt{2} + 1}{\sqrt{2}(\sqrt{2} + 1)} = \frac{2 + \sqrt{2}}{2 + \sqrt{2}} = 1$

S11. Ans.(b)

Sol.

Total surface area of cylinder = $2\pi rh + 2\pi r^2$

Curved surface area of cylinder = $2\pi rh$

Required ratio

	$2\pi rh + 2\pi r^2$	:	$2\pi rh$
$\Rightarrow$	$2\pi r(h + r)$	:	$2\pi rh$
$\Rightarrow$	$3.5 + 7.5$	:	7.5
$\Rightarrow$	11	:	7.5
	22	:	15

Required ratio = 22 : 15

S12. Ans.(a)
Sol.
Volume of right-circular cone = $\frac{1}{3}\pi r^2 h$
Let radius = r
Height = h
Radius increase 100% = $r + \frac{r \times 100}{100}$
$= 2r$
Height increase 100% = $h + \frac{h \times 100}{100}$
$= 2h$
New volume = $\frac{1}{3} \times \pi \times (2r)^2 \times 2h$
$= \frac{8}{3}\pi r^2 h$
% increase $= \left(\frac{\frac{8}{3}\pi r^2 h - \frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h}\right) \times 100$
$= 700\%$

S13. Ans.(d)
Sol.
$x^2 + (x+4)^2 = 656$
$\Rightarrow x^2 + x^2 + 16 + 8x = 656$
$\Rightarrow x^2 + 4x - 320 = 0$
$\Rightarrow x = 16$, ($x = -20$ is inadmissible)
∴ Sides of the square are x = 16 cm
And (x + 4) = 20 cm.

S14. Ans.(a)
Sol.
ATQ,
$2\pi r = 2 \times \frac{22}{7} \times 4.2 = 26.4$ meter
$26.4 = 2(6x + 5x)$
$6x = 7.2$ m

S15. Ans.(d)
Sol.
ATQ,
$(2\pi r - 2r) = 15$
$2r\left(\frac{22}{7} - 1\right) = 15 \Rightarrow r = \frac{7}{2}$
$\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38.5 \text{ m}^2$

S16. Ans.(b)

Sol.

$2(2x + x) \times 11 = 2640$

X=40m

so, the area of ceiling = $2x \times x = 3200\ m^2$

S17. Ans.(c)

Sol.

ATQ.

$\pi D \times 560 = 1.1 \times 1000$

$= \pi D \times 560 = 1100$

$\therefore D = \frac{110\times7}{56\times22}$

$= \frac{5}{8} m \Rightarrow \frac{5\times100}{8} \Rightarrow 62.5\ \text{cm}$

S18. Ans.(c)

Sol.

Let water is filled upto h height in the cylinder.

Let after dropping sphere in the cylinder height of water become h_1

Volume of water raised $(h_1 - h)$ = Volume of sphere

$\pi r^2(h_1 - h) = \frac{4}{3}\pi(3)^3$

$\pi 36 \times (h_1 - h) = \frac{4}{3} \times \pi \times 27$

$h_1 - h = 1$ cm

S19. Ans.(a)

Sol.

External radius, $r_2 = 10/2$

= 5 cm

Internal radius, $r_1 = \frac{10-2}{2}$

= 4 cm

Area to be painted

$= 2\pi r_1^2 + 2\pi r_2^2 + (\pi r_1^2 - \pi_2^2)$

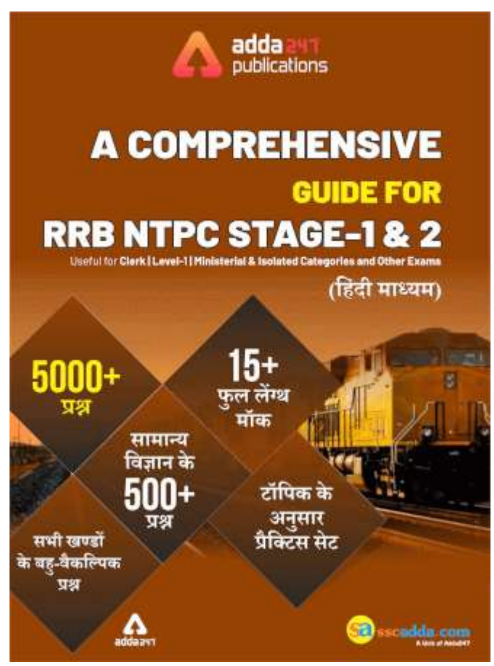
$= 3\pi r_1^2 + \pi r_2^2$

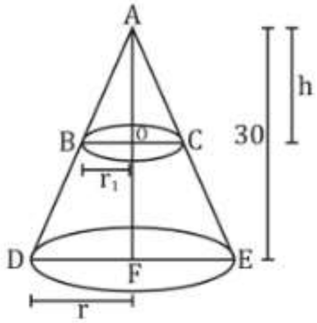
$= \pi\ (3 \times 25 + 16)$

$= \frac{22}{7} \times 91$

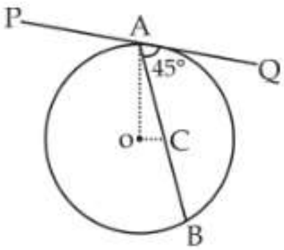
Cost = $\frac{22}{7} \times 91 \times 0.7$

= Rs. $200.2 \cong 200$ Rs.



S20. Ans.(c)
Sol.


$\Delta ADF \cong \Delta ABO$
$$\frac{h}{30} = \frac{r_1}{r}$$
$$r_1 = \frac{hr}{30}$$
Volume of smaller cone = $\frac{1}{27}$ × Volume cone larger cone
$$\frac{1}{3}\pi r_1^2 h = \frac{1}{3}\pi r^2 \times 30 \times \frac{1}{27}$$
$$r_1^2 h = r^2 \times 30 \times \frac{1}{27}$$
$$\frac{h^2 \times r^2 \times h}{900} = r^2 \times \frac{10}{9}$$
$h^3 = 1000$, h = 10 cm

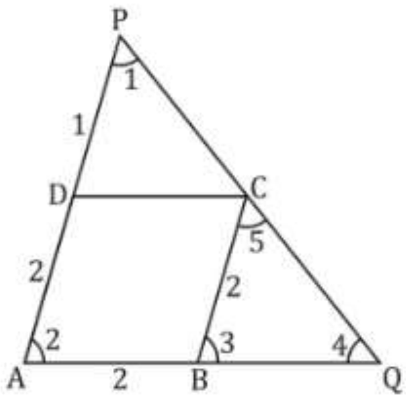
S21. Ans.(c)
Sol.


Let the center be O tangent be PQ and chord be AB
∵ ∠BAQ = 45° (Given)
∠OAQ = 90° (Angle between radius & tangent)
∴ ∠OAB = 45°
AB = 6 cm (Given)
AB = CB = 3 cm (⊥ drawn from center to chord bisect the chord)
In ΔOCA
∠OCA = 90°
∠OAC = 45°
∠AOC = 45°
∵ ∠OAC = ∠AOC = 45°
∴ AC = OC = 3 cm
$OA = \sqrt{AC^2 + OC^2}$ (Pythagorus theorem)
$OA = \sqrt{3^2 + 3^2} = \sqrt{9 + 9} = \sqrt{18}$
$OA = 3\sqrt{2}$ cm

S22. Ans.(a)

Sol. According to question,

Given:



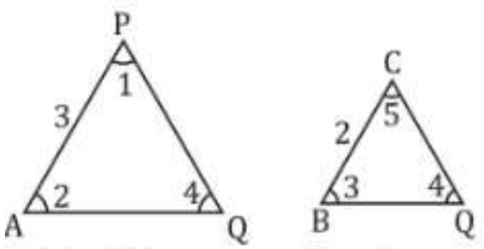
ABCD is a rhombus

AB = BC = CD = DA

$\Rightarrow DP = \frac{1}{2}AB \ \Rightarrow \frac{DP}{AB} = \frac{1}{2}$

In a rhombus ∠2 = ∠3

$\therefore \Delta APQ \sim \Delta BCQ$ (∵ ∠Q is common and ∠2 = ∠3)



$\Rightarrow \frac{AP}{BC} = \frac{AQ}{BQ}$ $\quad \frac{AQ}{BQ} = \frac{3}{2}$

$\Rightarrow \frac{AB+BQ}{BQ} = \frac{3}{2}$ $\quad$ (∴ AQ = AB + BQ)

$\Rightarrow \frac{AB}{BQ} + 1 = \frac{3}{2} \Rightarrow \frac{AB}{BQ} = \frac{3}{2} - 1 \Rightarrow \frac{AB}{BQ} = \frac{1}{2}$

$\Rightarrow \therefore \frac{BQ}{AB} = \frac{2}{1}$

S23. Ans.(b)

Sol. Inradius of equilateral triangle = $\frac{a}{2\sqrt{3}}$, (a is side)

Area of incircle $= \pi\left(\frac{a}{2\sqrt{3}}\right)^2$

$\pi\left(\frac{a}{2\sqrt{3}}\right)^2 = 36\pi$

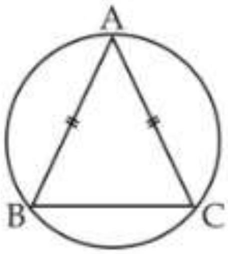
$\frac{a^2}{12} = 36$

$a = 12\sqrt{3}$

Circumradius of equilateral triangle = $\frac{a}{\sqrt{3}} = \frac{12\sqrt{3}}{\sqrt{3}} = 12$ cm

Area of Circumcircle = $\pi(12)^2 = 144\ \pi\ cm^2$

S24. Ans.(b)
Sol.



In radius of a triangle

$= \frac{abc}{4 \text{ Area of triangle}}$

Area of Isosceles triangle

$= \frac{1}{2} b \sqrt{a^2 - \frac{b^2}{4}}$

$b = 24, a = 12\sqrt{5}$

$\text{Area} = \frac{1}{2} \times 24 \sqrt{144 \times 5 - \frac{576}{4}}$

$= \frac{1}{2} \times 24\sqrt{144 \times 5 - 144}$

$= \frac{1}{2} \times 24 \times 12 \times 2$

$= 288$

$\text{Inradius} = \frac{24 \times 12\sqrt{5} \times 12\sqrt{5}}{4 \times 288}$

$= 15$ cm

S25. Ans.(a)
Sol.

Distance between two points

$= \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3 - a)^2 + (a + 3)^2}$

$= \sqrt{9 + a^2 - 6a + a^2 + 9 + 6a}$

$= \sqrt{2a^2 + 18}$

ATQ,

$\sqrt{2a^2 + 18} = 6$

$2a^2 + 18 = 36$

$2a^2 = 18$

$a^2 = 9$

$a = \pm 3$

S26. Ans.(a)

Sol.

According to question

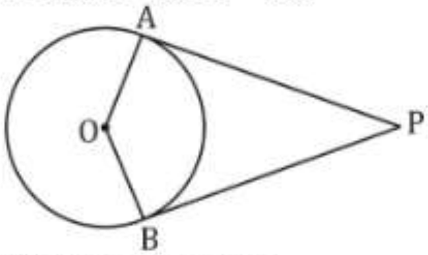
Given: PAOB is quadrilateral

∵ ∠AOB : ∠APB

5x : 1x

Note: In Quadrilateral sum of opposite angle is 180°

∴ ∠AOB + ∠APB = 180°



Then 5x + x = 180°

6x = 180°

x = 30°

∴ ∠APB = 30°

S27. Ans.(d)

Sol.

Slope of time = $\frac{y_1 - y_1}{x_2 - x_1}$

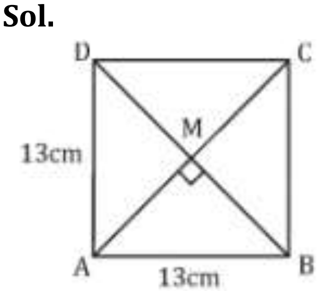
$$= \frac{\frac{4}{3} - 4}{3 - \frac{1}{2}}$$

$$= \frac{\frac{-8}{3}}{\frac{5}{2}}$$

$$= \frac{-16}{15}$$

S28. Ans.(d)

Sol.



Let BD = 24 cm

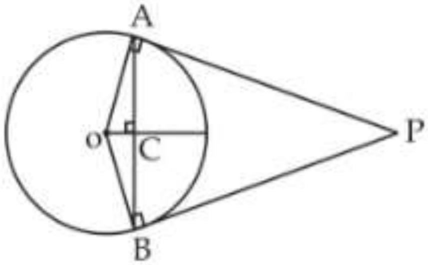
∴ BM = 12 cm

∴ AM = $\sqrt{13^2 - 12^2}$ = 5 cm

∴ AC = 2AM = 10 cm

S29. Ans.(a)

Sol.



$AC = \sqrt{OA^2 - AC^2}$

AC = 3

$AP^2 - CP^2 = 9 \ldots (i)$

and

$OP^2 = OA^2 + AP^2$

$OP^2 = 25 + 9 + CP^2$ [using equation (i)]

$(OC + CP)^2 = 34 + CP^2$

$16 + CP^2 + 8C = 34 + CP^2$

$CP = \frac{18}{8}$

$CP = \frac{9}{4}$

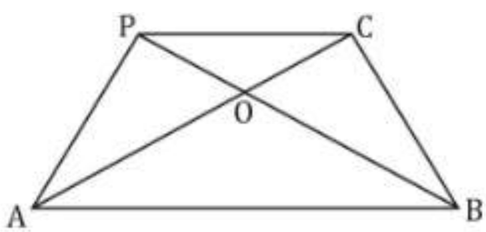
$\Rightarrow OP = OC + CP$

$OP = 4 + \frac{9}{4}$

$OP = \frac{25}{4}$

S30. Ans.(a)

Sol.



CD || AB

$\angle OAB = \angle OCD$ [alternate angles]

$\angle ODC = \angle OBA$ [alternate angles]

$\angle AOB = \angle DOC$

So,

$\Delta AOB \sim \Delta DOC$

$\frac{\text{area of } \Delta AOB}{\text{area of} \Delta DOC} = \frac{AB^2}{CD^2}$

= 4 : 1