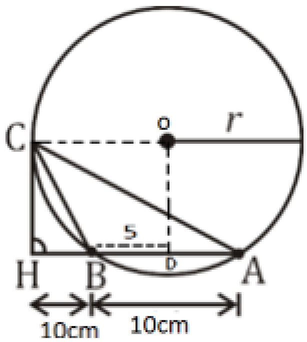
Solutions

S1. Ans.(c)
Sol.

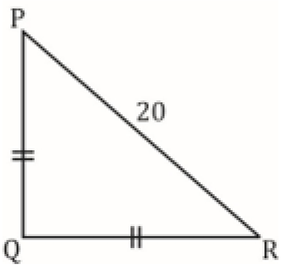


Let O be the centre of Circle
OC is perpendicular to CH and OD is also perpendicular to AB
So, we can say CHDO is a rectangle and DH= OC
D is the mid point of AB So DH = DB + BH
DH = 10+5= 15
CO = HD = r = 15cm

S2. Ans.(c)
Sol.
Δ OQP is right angled {radius OQ $\perp^r$ tangent QP}
$\therefore OP^2 = OQ^2 + QP^2$
$(25)^2 = (7)^2 + QP^2$
$\Rightarrow PQ = \sqrt{625 - 49} = 24$ cm

S3. Ans.(b)
Sol.
Two triangles are said to be similar,
If ratio of sides of both triangles is same i.e.,
$\frac{PQ}{DE} = \frac{2.5}{5} = \frac{1}{2}$
$\& \frac{QR}{EF} = \frac{2}{4} = \frac{1}{2}$
$\frac{PR}{DF} = \frac{3}{6} = \frac{1}{2}$
$\Rightarrow \frac{PQ}{DE} = \frac{QR}{EF} = \frac{PR}{DF}$
$\Delta PQR \sim \Delta DEF$

S4. Ans.(b)
Sol.



If PQ = QR, PR = 20

$PQ^2 + QR^2 = (20)^2$

$2PQ^2 = 400$

$PQ = 10\sqrt{2} = QR$

Area$=\frac{1}{2} \times QR \times PQ = \frac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2} = 100$

S5. Ans.(b)

Sol.

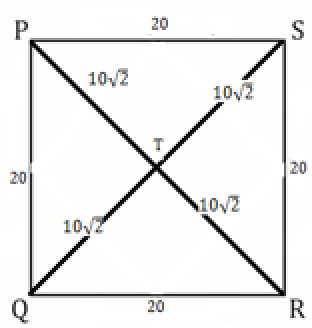
$\angle RQA = \angle RAQ = 30°$ & $\angle PQA = \angle PAQ = 55°$

So, $\angle PQR = 55° - 30° = 25°$

S6. Ans.(b)

Sol.

PQRS is a square of side 20 each so diameter $=20\sqrt{2}$



PQ = 20, PR$=20\sqrt{2}$

PT$=10\sqrt{2}$ TR$=10\sqrt{2}$

Perimeter of 1 triangle PTQ = $20 + 20\sqrt{2}$

Perimeter of 4 triangle = $4(20 + 20\sqrt{2}) = 80 + 80\sqrt{2}$

S7. Ans.(b)

Sol.

We have,

AB = 12 , BQ = 7

QC = RC = 5

BQ = BP =7

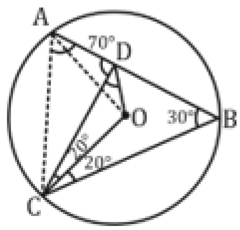
AP = 12 - 7 = 5

AP = AR = 5

AC = AR + RC = 5+5 =10

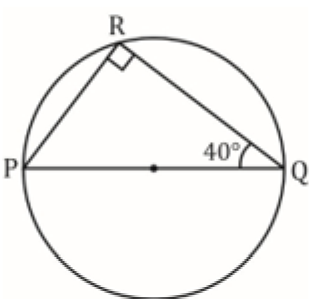
S8. Ans.(b)

Sol.



Join AO & AC
So,
∠AOC = 2 ∠ABC = 60
⇒ ∠ACO = ∠CAO = 60° (Since, AO = OC) ⇒ CA=AO=CO
∴ ACD = 40°
& ∠CAB = 180° - 80° - 30° = 70°
& ∠CDA = 180° - 70° - 40° = 70°
∴ CA = CD = CO
$\Rightarrow \angle CDO = \frac{(180-20)}{2} = 80$

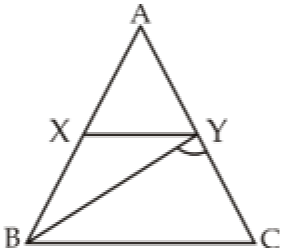
S9. Ans.(c)
Sol.
quadrilateral AOBP,
∠ PAO + ∠AOB + ∠OBP + ∠ APB = 360°
90° + 120° + 90° + ∠ APB = 360°
⇒∠APB = 60°

S10. Ans.(c)
Sol.


If PQ is diameter.
Angle at circumference of a circle is always 90°
So, ∠PRQ = 90 & ∠PQR=40(given)
Hence
∠QPR = (90 – 40) = 50°

S11. Ans.(b)
Sol.

Since,



XY || BC

We have

∠XYB = ∠YBC

{Alternate interior angels}

But,

∠XYB = ∠BYC

{Given by bisects angles XYC}

Hence, ∠BYC = ∠YBC

⇒ BC = CY

{Opposite angles at equal sides are equal in d triangles}

S12. Ans.(c)

Sol.

Clearly,

$2x^2 = 169 \times 2$

$\Rightarrow x = 13$

& In ΔACE & ΔFDE

∠E common & ∠FDE = ∠ACE

So,

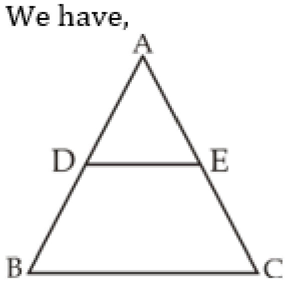
$$\frac{AC}{FD} = \frac{AE}{FE} \Rightarrow AC = \frac{2 \times (13\sqrt{2} + 1)}{1} = 26\sqrt{2} + 2$$

$\therefore AC + x = 26\sqrt{2} + 15$

S13. Ans.(d)

Sol.

We have,



AD = 3 cm, BD= 4 cm

& AE = 4.4 cm, DE = 6 cm

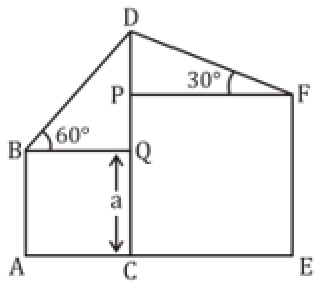
Since, ΔABC ~ ΔADE

$$\therefore \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{3}{7} = \frac{6}{BC} \Rightarrow BC = 14 \text{ cm}$$

S14. Ans.(b)

Sol.



$\frac{DQ}{BD} = \frac{\sqrt{3}}{2} \Rightarrow DQ = 3$ m

& $\frac{DP}{DF} = \frac{1}{2} \Rightarrow DP = 2$m

∴ PQ = 3 – 2 = 1 m.

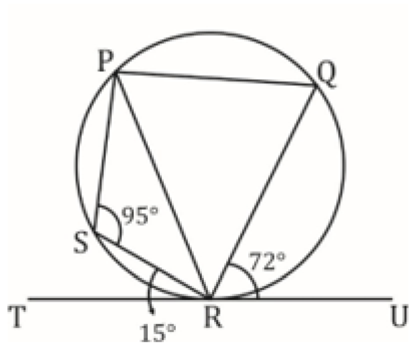
Let AB = QC = a m.

According to question

$\frac{CD}{EF} = \frac{5}{4}$

or $\frac{a+3}{a+1} = \frac{5}{4} \Rightarrow a = 7$ m

S15. Ans.(A)
Sol.



PQRS is a cyclic quadrilateral

sum of opposite angle = 180

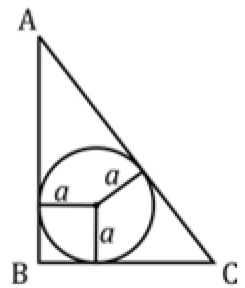
∠PSR + ∠PQR = 180

95 + ∠PQR = 180

∠PQR = 85

S16. Ans.(d)
Sol.

r – inradius of incircle of triangle



Perimeter = 15 cm (given)

∴ Semi-perimeter (S) = $\frac{15}{2}$ cm

Inradius of any triangle

$r \Rightarrow \frac{\Delta}{s}$ r = $\frac{\text{area}}{\text{semiperimeter}}$ where Δ is the area of triangle

∴ $r = 3$ cm given $3 \Rightarrow \frac{\text{area of triangle}}{\frac{15}{2}}$ $3 \times \frac{15}{2}$ = area of triangle

⇒ $\frac{45}{2}$ cm = area of triangle

∴ volume of prism ⇒ 270 cm³ (given) ∴ $270 = h \times \frac{45}{2} \Rightarrow$ h = 12 cm

S17. Ans.(a)
Sol.

Volume of bucket = $\frac{1}{3}\pi h\,(R^2 + r^2 + Rr) = \frac{1}{3} \times \frac{22}{7} \times 45(28^2 + 7^2 + 28 \times 7)$

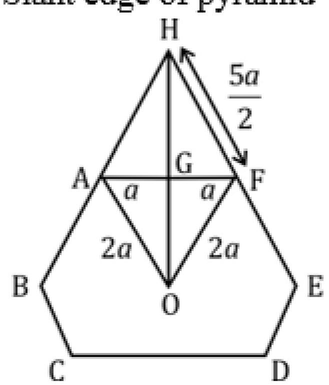
$= \frac{22}{7} \times 15 \times 1029 = 48510\text{ cm}^3$

S18. Ans.(c)
Sol.

Side of regular hexagon = $2a$cm

Area of hexagon = $6 \times \frac{\sqrt{3}}{4} \times (2a)^2 \Rightarrow \frac{6\sqrt{3}}{4} \times (2a)^2 \Rightarrow 6\sqrt{3}a^2$ cm²

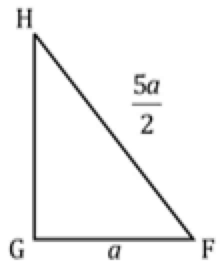
Slant edge of pyramid



⇒ $\frac{5a}{2}$ cm

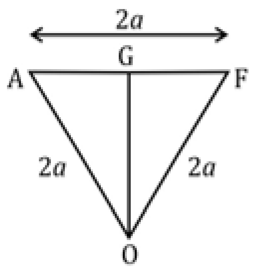
Slant edge ⇒ $\frac{5a}{2}$

(Given)



⇒ HF = $\frac{5a}{2}$ (slant edge) ⇒ HG = slant height (ℓ)⇒ GF = base ⇒ (a) (given)

Slant height ⇒ $\sqrt{\left(\frac{5a^2}{2}\right) - (a)^2} = \sqrt{\frac{25a^2}{4} - a^2} = \frac{\sqrt{21}a}{2}$



AOF is equilateral triangle of side 2a

∴ Altitude GO = $\frac{\sqrt{3}}{2} \times 2a = \sqrt{3}\,a$

∴ Slant height of the pyramid ⇒ $\sqrt{\left(\frac{\sqrt{21}a}{2}\right)^2 - \left(\sqrt{3}a\right)^2} = \sqrt{\frac{21}{4}a^2 - 3a^2} = \sqrt{\frac{9a^2}{4}} = \frac{3}{2}a$

∴ Volume of pyramid = $\frac{1}{3}$area of base × height = $\frac{1}{3} \times 6\sqrt{3}a^2 \times \frac{3}{2}a = 3\sqrt{3}a^3\text{cm}^3$

S19. Ans.(a)
Sol.

Volume of tetrahedron = $\frac{a^3}{6\sqrt{2}} = \frac{12^3}{6\sqrt{2}} = \frac{1728}{6\sqrt{2}}$

$= 144\sqrt{2}\text{ cm}^3$

S20. Ans.(a)
Sol.

Decrease in base radius = (Decrease in base area)$^{\frac{1}{2}} = \left(\frac{1}{9}\right)^{\frac{1}{2}}$

Let initial radius and height be 3r and h

∴ New radius and height are r and 6 h

old lateral surface area = $2 \times \pi \times 3r \times h$ = 6π rh

New lateral surface area = $2 \times \pi \times r \times 6h$= 12 π rh

Required factor = $\frac{12\pi rh}{6\pi rh} = 2$

S21. Ans.(c)

Sol.

Decrease in radius = 50%= $\frac{1}{2}$ Increase in height = 50%= $\frac{1 \rightarrow \text{Increment}}{2 \rightarrow \text{Original}}$

	Radius	Height	Volume
Original	2	2	$(2)^2 \times (2) = 8$
	↓ 50% decrease	↓ 50% Increase	↓ –5
New	1	3	$(1)^2 \times (3) = 3$

Reduction in volume $= \frac{5}{8} \times 100 = 62\frac{1}{2}\%$

S22. Ans.(a)

Sol.

Volume of coffee $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (4)^3 = \frac{128}{3}\pi \text{ cm}^3$

Volume of cone $= \frac{1}{3}\pi r^2 \times h = \frac{1}{3}\pi(8)^2 \times 16 = \frac{1024}{3}\pi$

∴ Required percentage $= \frac{\frac{1024}{3} - \frac{128}{3}}{\frac{1024}{3}} \times 100 = 87.5\%$

S23. Ans.(c)

Sol.

radius of cone = radius of cylinder

Height of cone = height of cylinder = h

$\frac{\text{curvaed surface area of cylinder}}{\text{curved surface area of cone}}$

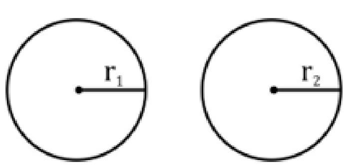
$\frac{2\pi rh}{\pi r\ell} = \frac{2\pi rh}{\pi r\ell} = \frac{8}{5} \Rightarrow \frac{h}{\ell} = \frac{4}{5} \Rightarrow \frac{h^2}{\ell^2} = \frac{16}{25} \Rightarrow 25\, h^2 = 16(h^2 + r^2)$

$\Rightarrow \frac{h^2}{r^2} = \frac{16}{9} = \frac{h}{r} = \frac{4}{3}$

∴ radius : height 3 : 4

S24. Ans.(a)

Sol.



Ratio of volume of sphere × ratio of weight per 1 cc. of material of each

= Ratio of weight of two sphere

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} \times \frac{289}{64} = \frac{8}{17}$$

$$\frac{r_1^3}{r_2^3} = \frac{8\times 64}{17\times 289} = \frac{8\times 8\times 8}{17\times 17\times 17}$$

$$\frac{r_1}{r_2} = \frac{8}{17} \Rightarrow 8 : 17$$

S25. Ans.(d)

Sol.

Radius of longer sphere = R units

Its volume = $\frac{4}{3}\pi R^3$

Now cones are formed with base radius and height same as the radius of larger sphere

∴ Volume of smaller cone = $\frac{1}{3}\pi R^3$

And one of the cone is converted into smaller sphere

Therefore volume of smaller sphere = $\frac{1}{3}\pi R^3$

$$\therefore \frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^3$$

$$\frac{r^3}{R^3} = \frac{1}{4}\ \frac{r}{R} = \frac{1}{\sqrt[3]{4}}$$

$$\therefore \frac{\text{Surface area of smaller sphere}}{\text{Surface area of larger sphere}}$$

$$\frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2}$$

$$\Rightarrow \frac{(1)^2}{\left((4)^{\frac{1}{3}}\right)^2} = \frac{(1)^2}{\left((2)^{\frac{2}{3}}\right)^2} = \frac{1}{2^{\frac{4}{3}}}$$