Solutions

S1. Ans.(b)

Sol. X = Set of all citizens of India

R = {(x, y) : x, y ∈ X, |x – y| = 5}

|x – x| = 0 ≠ 5 (R is not reflexive)

xRy ⇒ |x – y| = 5

⇒ |y – x| = 5 (R is symmetric)

xRy ⇒ |x – y| = 5

yRy ⇒ |y – z| = 5

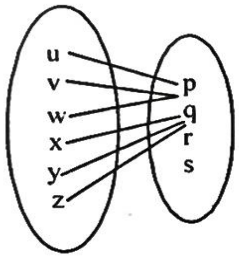
But |x – z| ≠ 5 (R is not transitive)

S2. Ans.(c)

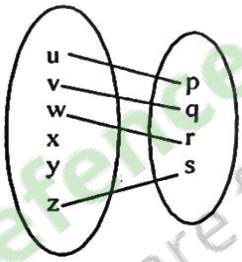
Sol. Given that, A = {u, v, x, y, z}; B = {p, q, r, s}

As we know, a mapping f : x → y is said to be a function, if each element in the set x has its image in set y. It is also possible that these are few elements in set y which are not the image of any element in set x. Every element in set x should have one and only one image.

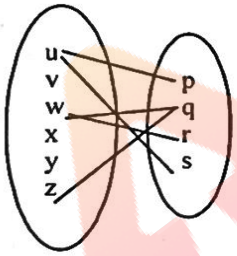
(i)



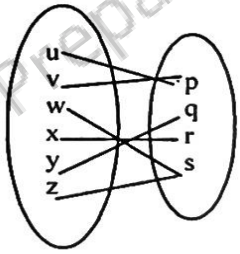
(ii)



(iii)



(iv)



(ii) and (iii) ae not functions.

S3. Ans.(c)

Sol. Given equation $ax^2 + bx + c = 0$ $(where\ a \neq 0)$

α and β are roots of given equation.

$(a\alpha + b)(a\beta + b) = a^2\ \alpha\beta + ab\alpha + ab\beta + b^2$

$= a^2\ \alpha\beta + ab(\alpha + \beta) + b^2$

From the given quadratic equation

$\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

$a^2 \times \frac{c}{a} + ab \times -\frac{b}{a} + b^2 = ac$

S4. Ans.(c)
Sol. S = Set of all integers and
R = {(a, b), a, b ϵ S and ab ≥ 0}
For reflexive : aRa ⇒ a.a = $a^2 \geq 0$
For all integers a. a ≥ 0
For symmetric : aRb ⇒ ab ≥ 0 ∀ a, b ϵ S
If ab ≥ 0, then ba ≥ 0 ⇒ bRa
For transitive:
If ab ≥ 0, bc ≥ 0, then also ac ≥ 0
Relation R is reflexive, symmetric and transitive.
Therefore relation is equivalence.

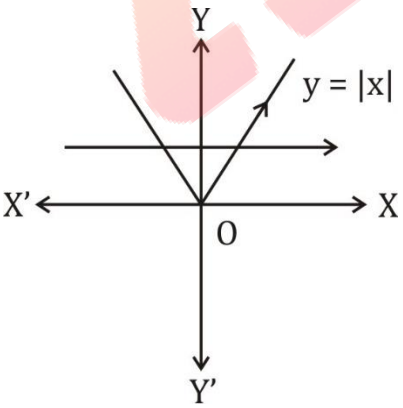
S5. Ans.(c)
Sol. We have, $2a^2\ x^2 - 2abx + b^2 = 0$
Discriminant, D = $(-2\ ab)^2 - 4\ (2a^2)(b^2)$
$= 4a^2b^2 - 8a^2b^2 = -4a^2b^2 < 0$
Roots are always complex.

S6. Ans.(a)
Sol.

2	40	
2	20	0
2	10	0
2	5	0
2	2	1
	1	0

$(11110)_2 = 2^4 \times 1 + 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 0 = 16 + 8 + 4 + 2 + 0 = 30$
$(1010)_2 = (2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 0 = 8 + 0 + 2 + 0) = 10$
Sum = 30 + 10 = 40 = $(101000)_2$

S7. Ans.(d)
Sol.



$f: Z \to N\ and\ f(x) = |x|$
When we draw a parallel line to x-axis. It cuts the curve into more than one point.
Therefore, $f(x) = |x|$ is not one-one.

S8. Ans.(d)

Sol. The two complex numbers are

P = x + iy and Q = α + iβ

Quotient = $\frac{P}{Q} = \frac{x+iy}{\alpha+i\beta}, \left|\frac{P}{Q}\right| = \left|\frac{x+iy}{\alpha+i\beta}\right| = \frac{\sqrt{x^2+y^2}}{\sqrt{\alpha^2+\beta^2}} = \sqrt{\frac{x^2+y^2}{\alpha^2+\beta^2}} = \left|\frac{Z_1}{Z_2}\right| = \left|\frac{Z_1}{Z_2}\right|$

Hence, the quotient of their modulus is equal to the quotient of their moduli.

S9. Ans.(a)

Sol.

$|2z-1| = |z-2|$

$|2(x+iy)-1| = |x+iy-2|$

$|(2x-1)+2yi| = |(x-2)+iy|$

$\sqrt{(2x-1)^2+y^2} = \sqrt{(x-2)^2+y^2}$

Squaring both sides

$4x^2+1-4x+4y^2 = x^2+4-4x+y^2$

$\Rightarrow 3x^2+3y^2 = 3$

$\Rightarrow x^2+y^2 = 1$

It is equation of a circle.

∴ The point z describes a circle.

S10. Ans.(c)

Sol. GP = x

$\frac{a}{1-r}$ = x (where, a = 1st term and r = common ratio)

$\Rightarrow \frac{2}{1-r} = x$ ….(i) (∵ Given a = 2 and | r | < 1)

⇒ –1 < r < 1 ⇒ 1 > – r > – 1

⇒ 1 + 1 > 1 – r > 1 – 1

⇒ 0 < 1 – r < 2.

$\Rightarrow \frac{1}{1-r} > \frac{1}{2}, \frac{2}{1-r} > 1$

From equation (i) x > 1

Hence, 1 < x < ∞.

S11. Ans.(c)

Sol. Total number of balls = 5

Number of black balls = 2

Required probability = $\frac{n(E)}{n(S)} = \frac{{}^3C_0 \times {}^2C_2}{{}^5C_2} = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$

S12. Ans.(c)

Sol. r = $\sqrt{b_{xy}.b_{yx}} = \sqrt{\left(-\frac{1}{6}\right) \times \left(-\frac{3}{2}\right)} = \sqrt{\frac{1}{2} \times \frac{1}{2}} = \pm\frac{1}{2}$

b_{xy} *and* b_{yx} both have negative sign.

Therefore we have to take negative sign

Hence, correlation coefficient (r) = $-\frac{1}{2}$.

S13. Ans.(c)

Sol. I: Variance is not dependent on change of origin.

Therefore, if every x_i is increased by 2, the variance of the new set of numbers is not changed.

II: Variance is dependent on change of scale.

If the number x_i is squared the variance of the new set is V^2

$V_{xi} \times V_{xi} = V.V = V^2$

S14. Ans.(b)

Sol. Mean of the squares of the first 20 natural number = $\frac{(n+1)(2n+1)}{6} = \frac{21\times41}{6} = 143.5$

S15. Ans.(c)

Sol. According to question p + q + r + s + t = 15 + 20 = 35

Average p, q, r, s and t = $\frac{35}{5} = 7$

S16. Ans.(a)

Sol. The cumulative frequency of the largest observed value must always be less then the total number of observations.

S17. Ans.(b)

Sol. P(A) $= \frac{6}{12} = \frac{1}{2}$, P(B) $= \frac{4}{12} = \frac{1}{3}$

Req. probability = $\frac{1}{2}.\frac{1}{3}.\frac{1}{2} + \frac{1}{3}.\frac{1}{2}.\frac{1}{3} = \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$

S18. Ans.(a)

Sol. Number of words = $5! \times {}^7C_3 \times {}^4C_2 = 120 \times \frac{7!}{4!3!} \times \frac{4!}{2!2!} = 25200$

S19. Ans.(c)

Sol. Statement I:

n (X) = 2

p = $\frac{n(X)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Q = 1 – p = $1 - \frac{1}{3} = \frac{2}{3}$

arithmetic mean of X = np = $18 \times \frac{1}{3} = 6$

Statement 2 : Standard deviation of

X = $\sqrt{\text{variance of X}} = \sqrt{18 \times \frac{1}{3} \times \frac{2}{3}} = \sqrt{4} = 2$

Hence, statements 1 and 2 both are correct.

S20. Ans.(c)

Sol. As 'A' must be first letter of each word.

Total number of words = 4! = 24

S21. Ans.(a)

Sol. $3, \sqrt{3}, 1, \frac{1}{\sqrt{3}}, \ldots\ldots, \infty$

This is a Geometric Progression with a = 3, r = $\frac{1}{\sqrt{3}}$.

$S_\infty = \frac{a}{1-r} = \frac{3}{1-\frac{1}{\sqrt{3}}} = \frac{3\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{3\sqrt{3}(\sqrt{3}+1)}{2}$

S22. Ans.(a)

Sol. Let z = x + iy, $\overline{z} = x - iy$

$|z + \overline{z}| = |z - \overline{z}|$

$|(x + iy) + (x - iy)| = |(x + iy) - (x - iy)|$

$|2x| = |2y|$

x = ± y

S23. Ans.(b)

Sol.

2	251	1
2	125	1
2	62	0
2	31	1
2	15	1
2	7	1
2	3	1
	1	

Therefore, $(251)_{10} = (11111011)_2$

S24. Ans.(d)

Sol.

$\frac{(1+i)(2+i)}{3-i} = \frac{1+3i}{3-i} = \frac{1+3i}{3-i} \times \frac{3+i}{3+i} = \frac{10i}{10} = i \; or \; 0 + i$

Argument, $\theta = \tan^{-1}\left(\frac{1}{0}\right) = \tan^{-1}(\tan\frac{\pi}{2}) = \frac{\pi}{2}$

S25. Ans.(a)

Sol.

$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = -A$

Hence, A is skew symmetric matrix

$|A| = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix} = 1(-6) - 2(-3) = -6 + 6 = 0$

Therefore, A is non-invertible.