Solutions

S1. Ans.(a)

Sol. Given

tan 19° & tan 26° are the roots of $x^2 + px + q = 0$

$\Rightarrow$ tan 19° + tan 26° = – p

tan 19° tan 26° = q

Consider,

$\tan(19° + 26°) = \frac{\tan 19° + \tan 26°}{1 - \tan 19° \tan 26°}$

$\tan(45°) = \frac{-p}{1-q}$

$\Rightarrow 1 = \frac{-p}{1-q}$

$\Rightarrow$ 1 – q = – p

$\Rightarrow 1 = q - p$

S2. Ans.(b)

Sol. We know,

n^{th} term = sum upto n terms – sum upto (n – 1) terms.

$a + (n-1)d = n(n+1) - (n-1)n$

$a + (n-1)d = n[n+1-n+1]$

$a + (n-1)d = 2n$

For n = 4

a + 3d = 8.

S3. Ans.(a)

Sol. $(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2 - \sec^2\alpha \sec^2\beta$.

$= \left(\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta}\right)^2 + \left(\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}\right)^2 - \frac{1}{\cos^2\alpha\cos^2\beta}$

$= \frac{(\cos\alpha\cos\beta + \sin\alpha\sin\beta)^2}{\cos^2\alpha\cos^2\beta} + \frac{(\sin\alpha\cos\beta - \sin\beta\cos\alpha)^2}{\cos^2\alpha\cos^2\beta} - \frac{1}{\cos^2\alpha\cos^2\beta}$

$= \frac{\cos^2(\alpha-\beta)}{\cos^2\alpha\cos^2\beta} + \frac{\sin^2(\alpha-\beta)}{\cos^2\alpha\cos^2\beta} - \frac{1}{\cos^2\alpha\cos^2\beta}$

$= \frac{\cos^2(\alpha-\beta) + \sin^2(\alpha-\beta) - 1}{\cos^2\alpha\cos^2\beta}$

$= \frac{1-1}{\cos^2\alpha\cos^2\beta} = 0$

S4. Ans.(b)

Sol. P = cosec θ – cot θ

$q = (\text{cosec}\,\theta + \cot\theta)^{-1}$

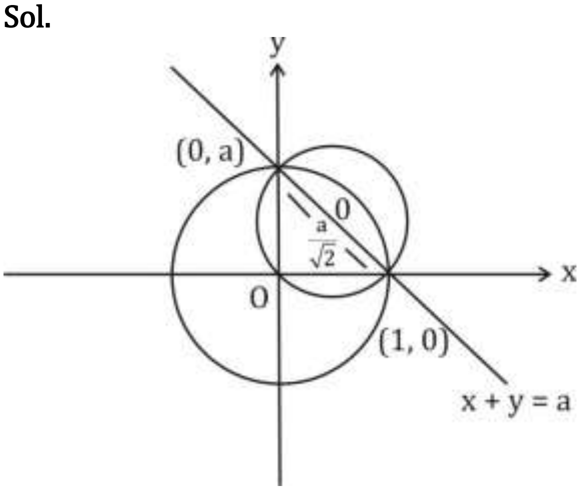
$= \frac{1}{(cosc\theta + \cot\theta)}$

$= \frac{1}{(cosec\,\theta + \cot\theta)} \times \frac{(cosec\,\theta - \cot\theta)}{(cosec\,\theta - \cot\theta)}$

$= \frac{cosec\,\theta - \cot\theta}{cosec^2\theta - \cot^2\theta} = cosec\,\theta - \cot\theta = p$

S5. Ans.(c)
Sol. ∠A : ∠B : ∠C = 1 : 2 : 3
We know, ∠A + ∠B + ∠C = 180°
x + 2x + 3x = 180°
6x = 180°
x = 30°
⇒ ∠ A = 30°
∠ B = 60°
∠ C = 90°
By sine Rule
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$
$\Rightarrow \frac{a}{\sin 30°} = \frac{b}{\sin 60°} = \frac{c}{\sin 90°} = k$
$\Rightarrow \frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{1} = k$
$\Rightarrow \frac{a}{1} = \frac{b}{\sqrt{3}} = \frac{c}{2} = \frac{1}{2}$
⇒ a : b : c = 1 : $\sqrt{3}$: 2

S6. Ans.(d)
Sol.
1. d = $\left|\frac{\alpha\cos\theta+\beta\sin\theta-p}{\sqrt{\cos^2\theta+\sin^2\theta}}\right|$
= |α cos θ + β sin θ – P|
2. line $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow bx + ay - ab = 0$
d = $\left|\frac{\alpha b+a\beta-ab}{\sqrt{b^2+a^2}}\right|$

S7. Ans.(b)
Sol.


O' = $\frac{0+a}{2}, \frac{a+0}{2} = \left(\frac{a}{2}, \frac{a}{2}\right)$
Equation of the circle whose centre is $\frac{a}{2}, \frac{a}{2}$ and radius r = $\sqrt{\left(a-\frac{a}{2}\right)^2 + \left(0-\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$ is
$\left(x-\frac{a}{2}\right)^2 + \left(y-\frac{a}{z}\right)^2 = \left(\frac{a}{\sqrt{2}}\right)^2$
$\Rightarrow x^2 + y^2 - ax - ay = 0$

S8. Ans.(b)

S9. Ans.(d)

Sol. $2x^2 - 3y^2 - 6 = 0$

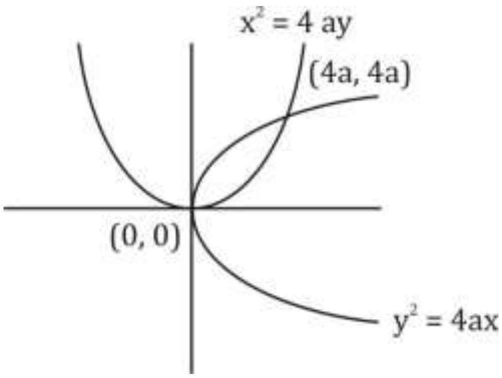
$2x^2 - 3y^2 = 6$

$\frac{2x^2}{6} - \frac{3y^2}{6} = 1$

$\frac{x^2}{3} - \frac{y^2}{2} = 1$

S10. Ans.(a)

Sol.



S11. Ans.(d)

Sol. We know,

The diagonals of a rectangle bisect each other.

i.e. the mid point of (1, 3) and (5, 1) lic on the line y = 2x + C.

$\Rightarrow 0 = \left(\frac{1+5}{2}, \frac{3+1}{2}\right) = (3, 2)$

∵ (3, 2) lie on the line y = 2x + c

⇒ C = – 4

S12. Ans.(c)

Sol. If the lines $4x + 3y - 1 = 0, x - y + 5 = 0, bx + 5y - 3 = 0$ are concurrent, then.

$$\begin{vmatrix} 4 & 3 & -1 \\ 1 & -1 & 5 \\ b & 5 & -3 \end{vmatrix} = 0$$

⇒ b = 6

S13. Ans.(b)

Sol. The slope of the straight line which is perpendicular to x = y (here $m_1 = 1$) is –1

As $m_1 \, m_2 = -1$

$1 \times m_2 = -1$

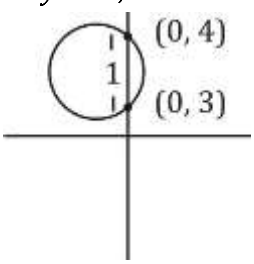
$m_2 = -1$

The equation of line whose slope is –1 & passing through (3, 2) is

(y – 2) = –1 (x – 3)

y + x = 5.

S14. Ans.(a)
Sol. By Solving these three lines x + y – 4 = 0, 3x + y – 4 = 0, and x + 3y – 4 = 0 we get three intersection points
i.e. $A = (0, 4), B = (1, 1), C = (4, 0)$
$\Rightarrow AB = \sqrt{10}$
$BC = \sqrt{10}$
$AC = \sqrt{32}$

S15. Ans.(a)
Sol. Put x = 0
$y^2 - 7y + 12 = 0$
$\Rightarrow y = 4, 3$


Y intercept = 4 – 3 = 1

S16. Ans.(a)
Sol.
$$\frac{\sin 34° \cos 236° - \sin 56° \sin 124°}{\cos 28° \cos 88° + \cos 178° \sin 208°}$$
$$\frac{\sin(90° - 56°) \cos(360° - 124°) - \sin 56° \sin 124°}{\cos 28° \cos 88° + \cos(90° + 88°) \sin(180° + 28°)}$$
$$\frac{\cos 56° \cos 124° - \sin 56° \sin 124°}{\cos 28° \cos 88° + (-\sin 88°)(-\sin 28°)}$$
$$\frac{\cos 56° \cos 124° - \sin 56° \sin 124°}{\cos 28° \cos 88° + \sin 88° \sin 28°}$$
$$\frac{\cos(56 + 124°)}{\cos(88 - 28)}$$
$$\frac{\cos(180)}{\cos(60°)} = \frac{-1}{1/2} = -2$$

S17. Ans.(c)
Sol. $\tan(54°)$
$= \tan(45° + 9°)$
$$= \frac{\tan 45° + \tan 9°}{1 - \tan 45° \tan 9°}$$
$$= \frac{1 + \tan 9°}{1 - \tan 9°}$$
$$= \frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$$

For Solutions (18-20):

Consider

$P^2 + 4pq + q^2 = Ax^2 + By^2$

$(x\cos\theta - y\sin\theta)^2 + 4(x\cos\theta - y\sin\theta)(x\sin\theta + y\cos\theta) + (x\sin\theta + y\cos\theta)^2 = Ax^2 + By^2$

$x^2\cos^2\theta + y^2\sin^2\theta - 2xy\cos\theta\sin\theta + 4\,x^2\sin\theta\cos\theta - 4y^2\sin\theta\cos\theta + 4\times y(\cos^2\theta - \sin^2\theta) = Ax^2 + By^2$

$x^2 + y^2 + 4x^2\sin\theta\cos\theta - 4y^2\sin\theta\cos\theta + 4\times 4(\cos^2\theta - \sin^2\theta) = Ax^2 + By^2$

By compare both sides, we get

A = 1 + 4 sin θ cos θ _______(1)

B = 1 – 4 sin θ cos θ _______(2)

4 × 4 [cos²θ – sin²θ] = 0 ________(3)

$\Rightarrow \cos^2\theta - \sin^2\theta = 1$

$\Rightarrow \tan^2\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

Put the value of $\theta = \frac{\pi}{4}$ in equation (1) & (2)

$A = 1 + 4.\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} = 3$

$B = 1 - 4.\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} = -1$

S18. Ans.(c)

S19. Ans.(b)

S20. Ans.(a)

S21. Ans.(a)

Sol. cos (α – β)

= cos [(θ – β) – cos (θ – α)]

= cos (θ – β) cos (θ – α) + sin (θ – β) sin (θ – α)

$= ab + \sqrt{1-\cos^2(\theta-\beta)}\ .\sqrt{1-\cos^2(\theta-\alpha)}$

$= ab + \sqrt{1-a^2}\,\sqrt{1-b^2}$

S22. Ans.(a)

Sol. $\sin^2(\alpha-\beta) + 2ab\cos(\alpha-\beta)$

$1 - \cos^2(\alpha-\beta) + 2ab\cos(\alpha-\beta)$

Now from Q 21.

$= 1 - \left[ab + \sqrt{1-a^2}\,\sqrt{1-b^2}\right]^2 + 2ab\left[ab + \sqrt{1-a^2}\,\sqrt{1-b^2}\right.$

$= 1 + a^2b^2 - (1-a^2)(1-b^2)$

$= 1 + a^2b^2 - 1 + a^2 + b^2 - a^2b^2$

$= a^2 + b^2$

S23. Ans.(c)

Sol. sinα + cosα = p

squaring both sides

$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = p^2$

⇒ $2\sin\alpha\cos\alpha = p^2 - 1$ ________(1)

Now consider,

$\cos^2(2\alpha)$

$= [\cos^2\alpha - \sin^2\alpha]^2$

$= (\cos\alpha + \sin\alpha)^2(\cos\alpha - \sin\alpha)^2$

$= p^2[\cos^2\alpha + \sin^2\theta - 2\sin\alpha\cos\alpha]$

$= p^2[1 - p^2 + 1]$

$= p^2[2 - p^2]$

S24. Ans.(d)

Sol.

$\sin^{-1}\frac{4}{5} + \sec^{-1}\frac{5}{4} - \frac{\pi}{2}$

$= \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5} - \frac{\pi}{2}$

$= \frac{\pi}{2} - \frac{\pi}{2}$

$= 0$

S25. Ans.(b)

Sol.

$\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\frac{1-q^2}{1+q^2} = \tan^{-1}\frac{2x}{1-x^2}$

$2\tan^{-1}p - 2\tan^{-1}q = 2\tan^{-1}x$

$\tan^{-1}p - \tan^{-1}q = \tan^{-1}x$

$= \tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1}x$ P > 0, q > 0

$= \frac{p-q}{1+pq} = x$

S26. Ans.(c)

Sol. Given, $\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\frac{1}{2}$

& $\tan\phi = \frac{1}{3} \Rightarrow \phi = \tan^{-1}\frac{1}{3}$

Now consider,

$(\theta + \phi) = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}\right)$ if $\frac{1}{2}.\frac{1}{3} < 1$

$= \tan^{-1}\left(\frac{5}{5}\right)$

$= \tan^{-1}(1)$

$= \tan^{-1}(\tan\frac{\pi}{4})$

$= \frac{\pi}{4}$

S27. Ans.(b)

Sol. Given cos A $= \frac{3}{4}$

$\Rightarrow 1 - 2\sin^2\frac{A}{2} = \frac{3}{4}$

$\Rightarrow \sin\frac{A}{2} = \frac{1}{\sqrt{8}}$

Now, $\sin\left(\frac{A}{2}\right)\sin\left(3\frac{A}{2}\right)$

$\text{Sin}\frac{A}{2}[3\sin\frac{A}{2} - 4\sin^3\frac{A}{2}]$ $[\because \sin 3x = 3\sin x - 4\sin^3 x]$

$= \frac{1}{\sqrt{8}}\left[\frac{3}{\sqrt{8}} - \frac{4}{8\sqrt{8}}\right]$

$= \frac{3}{8} - \frac{4}{64}$

$= \frac{24-4}{64} = \frac{20}{64} = \frac{5}{16}$

S28. Ans.(b)

Sol.

$\because \tan 75° = \tan(45° + 30°)$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow \cot 75° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

Now,

$\tan 75° + \cot 75°$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}{3-1}$

$= \frac{3+1+2\sqrt{3}+3+1-2\sqrt{3}}{2}$

$= \frac{8}{2} = 4$

S29. Ans.(b)

Sol. $\cos 46° \cos 47° \cos 48° \ldots\ldots \cos 90° \ldots\ldots \cos 135°$

$\cos 46° \cos 47° \cos 48° \ldots\ldots \cos 89° \times 0 \times \cos 91° \ldots\ldots \cos 135° = 0$

S30. Ans.(b)

Sol. $\sin 2\theta = \cos 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$2\sin\theta = 4\cos^2\theta - 3$

$2\sin\theta = 4 - 4\sin^2\theta - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0$

$\sin\theta = \frac{-2\pm\sqrt{4+16}}{8}$

$= \frac{-2\pm\sqrt{20}}{8}$

$= \frac{-1\pm\sqrt{5}}{4}$

S31. Ans.(b)

S32. Ans.(b)

S33. Ans.(b)

S34. Ans.(d)

S35. Ans.(c)
Sol. ${}^{10}C_5 - {}^{6}C_5$
$= \frac{10!}{5!5!} - \frac{6!}{5!1!}$
$= 252 - 6 = 246$

S36. Ans.(b)
Sol. $[(2x - 3y)^2]^2(2x + 3y)^2]^2$
$[(2x - 3y)(2x + 3y)]^4$
$[4x^2 - 9y^2]^4$.
Number of terms $= 4 + 1 = 5$.

S37. Ans.(b)
Sol. Given
${}^{n}C_0(ax)^0 = 1$
${}^{n}C_1(ax)^1 = 12x$
$n \times ax = 12x$
na = 12 ______(1)
and
${}^{n}C_2(ax)^2 = 64x^2$
$\frac{n(n-1)}{2}a^2x^2 = 64x^2$
$\frac{n(n-1)}{2}a^2 = 64$
$\frac{n(n-1)}{2} \times \frac{144}{n^2} = 64\ [from\ (1)]$
$\left(1 - \frac{1}{n}\right) = \frac{64}{72} = \frac{8}{9} = 1 - \frac{1}{9}$
$\Rightarrow n = 9$

S38. Ans.(c)

S39. Ans.(c)
Sol. Sum of $(p + q)^{th}\ term$ & $(p - q)^{th}\ term$ is
$(p + q)^{th}term + (p - q)^{th}term$
$a + (p + q - 1)d + a + (p - q - 1)d$
$a + pd + qd - d + a + pd - qd - d$
$[a + (p - 1)d] + [a + (p - 1)d]$
$2[a + (p - 1)d]$
$=$ Twice the p^{th} term.

S40. Ans.(b)

S41. Ans.(d)

Sol. $25\, cosec^2 x + 36 \sec^2 x$

$25(1 + \cot^2 x) + 36(1 + \tan^2 x)$

$61 + 25 \cot^2 x + 36 \tan^2 x$

$61 + \frac{25}{\tan^2 x} + 36 \tan^2 x$

∵ we know

A.M ≥ G.M.

$$\frac{\frac{25}{\tan^2 x} + 36 \tan^2 x}{2} \geq \sqrt{\frac{25}{\tan^2 x} \times 36 \tan^2 x}$$

$\frac{25}{\tan^2 x} + 36 \tan^2 x \geq 60$

$61 + \frac{25}{\tan^2 x} + 36 \tan^2 x \geq 61 + 60 = 121$

S42. Ans.(a)

Sol. $\det(2\, AB)$

$= 2^3 \det A \det B$

$= 8 \times 4 \times 3$

$= 96$

S43. Ans.(c)

Sol. $\det(3A\, B^{-1})$

$= 3^3 \det A \det(B^{-1})$

$= 27 \frac{\det A}{\det B}$

$= \frac{27 \times 4}{3} = 36$

For Solutions (44-45):

$Z = \frac{1+2i}{1-(1-i)^2}$

$= \frac{1+2i}{1-1-i^2+2i}$

$= \frac{1+2i}{1+2i}$

$= 1 = 1 + 0i$

S44. Ans.(c)

Sol. $|z| = \sqrt{x^2 + y^2}$

$= \sqrt{1^2 + 0^2}$

$= 1.$

S45. Ans.(a)

Sol. $\theta = \tan^{-1} \frac{y}{x}$

$= \tan^{-1} \frac{0}{1}$

$= \tan^{-1} 0$

$= 0.$

S46. Ans.(d)
Sol. $(-1)^{n-1}\ 5^{n+1}$

S47. Ans.(b)

S48. Ans.(d)

S49. Ans.(b)
Sol. A = {x ϵ z : $x^3 - 1 = 0$}
i.e. A = $\left\{x\ \epsilon\ z\ ; x = 1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$
and B = $\{x\ \epsilon\ z\ :\ x^2 + x + 1 = 0\}$
B = $\left\{x\ \epsilon\ z\ :\ x = \frac{-1-\sqrt{3}i}{2}, \frac{-1+\sqrt{3}i}{2}\right\}$
Hence A ∩ B = $\left\{\frac{-1-\sqrt{3}i}{2}, \frac{-1+\sqrt{3}i}{2}\right\}$

S50. Ans.(a)

S51. Ans.(d)

S52. Ans.(a)
Sol. $a_{11} = (-1)^{1+1} \times 0 = 0$
$a_{12} = (-1)^{1+2} \times 0 = 0$
$a_{13} = (-1)^{1+3}(-2) = -2$
$a_{21} = 0$
$a_{22} = 0$
$a_{23} = -1$
$a_{31} = 0$
$a_{32} = 0$
$a_{33} = 8$
$adj\ B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & -1 & 8 \end{bmatrix}$

S53. Ans.(d)
Sol. $|x^2 - x - 6| = x + 2$
$\Rightarrow x^2 - x - 6 = x + 2\ and - x^2 + x + 6 = x + 2$
First solving $x^2 - x - 6 = x + 2$
⇒ x = –2, 4 _____(1)
Now solve $-x^2 + x + 6 = x + 2$
⇒ x = + 2, –2 _______(2)
From (1) & (2)
x = 4, 2, – 2

S54. Ans.(b)
Sol.
$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
= I
Hence matrix A is an Involutory matrix.

S55. Ans.(a)
Sol.
$\begin{bmatrix} x & -3i & 1 \\ y & 1 & i \\ 0 & 2i & -i \end{bmatrix} = 6 + 11\text{i}$
$x(-i+2) + 3i(-yi) + 1(2iy) = 6 + 11i$
$-xi + 2x + 3y + 2iy = 6 + 11i$
By comparison both sides, we get
$2x + 3y = 6$ ______(1)
And $-x + 2y = 11$ _________(2)
By Solving (1) & (2), we get
x = –3
and y = 4

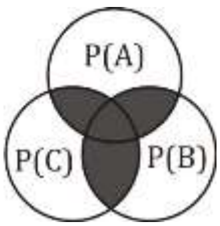
S56. Ans.(d)
Sol. Given,
$z^3 + 2z^2 + 2z + 1 = 0$ _____(1)
$z^{2017} + z^{2018} + 1 = 0$ _____(2)
New according to options there are four roots i.e. 1, –1, w, w^2
But 1 and –1 are not satisfying the given equations. Hence w, w^2 are the required roots.

S57. Ans.(b)
Sol. $c(20, n+2) = C(20, n-2)$
$\Rightarrow {}^{20}C_{n+2} = {}^{20}C_{n-2}$
$\Rightarrow n + 2 + n - 2 = 20$
$\Rightarrow 2n = 20$
⇒ n = 10

S58. Ans.(b)
Sol. A straight line can be draw by joining two points.
Hence the number of possible straight lines is
${}^{10}C_2$
$= \frac{10!}{8!\,2!}$
$= \frac{10 \times 9 \times 8!}{8! \times 2!}$
= 45

S59. Ans.(b)
Sol. Given
$a + b = -\frac{q}{p}$
$ab = \frac{r}{p}$
Here sum is negative & product is positive this is possible only when both the roots are negative
i.e. $a < 0, b < 0$.

S60. Ans.(b)
Sol. Let A be a set, then the of all the possible subsets of is called the power set of A and is denoted by P(A).

S61. Ans.(c)
Sol.


$A(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$

S62. Ans.(c)

S63. Ans.(b)
Sol. Given $P(A \cup B) = \frac{2}{3}$ & $P(A \cap B) = \frac{1}{6}$
$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow \frac{2}{3} = P(A) + P(B) - \frac{1}{6}$
$\Rightarrow \frac{2}{3} + \frac{1}{6} = P(A) + P(B)$
$\Rightarrow \frac{5}{6} = P(A) + P(B)$ _____(1)
Also, given $P(A \cap B) = \frac{1}{6}$
$P(A)\,P(B) = \frac{1}{6}$ ________(2)
From (1) & (2)
P(A) or $P(B) = \frac{1}{2}$ or $\frac{1}{3}$
Given $P(B) < P(A)$
$\Rightarrow P(B) = \frac{1}{3}$

S64. Ans.(c)
Sol. Given,
$x = 50$
Now the new mean $= \frac{50-5}{4} = \frac{45}{4} = 11.25$
Given $\sigma = 10$
Now the new standard duration $= \frac{10}{4} = 2.5$
[as addition and subtraction does not effect σ]

S65. Ans.(c)
Sol. Sample space S = {(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)}
Probability (that the first dice lands on 6) $= \frac{1}{5}$

S66. Ans.(c)
Sol.
$$\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}$$
$$\frac{4+4+1+1}{6\times 6}=\frac{10}{36}=\frac{5}{18}$$

S67. Ans.(b)
Sol.
$\frac{3}{n}\times\frac{2}{n-1}=\frac{1}{2}$
$12 = n^2 - n$
$0 = n^2 - n - 12$
$(n^2 - 4n) + (3n - 12) = 0$
$n(n-4) + 3(n-4) = 0$
$(n-4)(n+3) = 0$
$n = 4, -3$

S68. Ans.(a)
Sol. $\frac{52}{52}\times\frac{3}{51}\Rightarrow\frac{1}{17}$

S69. Ans.(b)
Sol. $P = \frac{1}{6}+\frac{1}{6}=\frac{1}{3}$
$\Rightarrow q = 1-\frac{1}{3}=\frac{2}{3}$
Given n = 8.
We know mean = np $= 8\times\frac{1}{3}=\frac{8}{3}$
And $\sigma = \sqrt{npq} = \sqrt{8\times\frac{1}{3}\times\frac{2}{3}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

S70. Ans.(b)
Sol. Given $P(\overline{A}\cap\overline{B}) = 0$
$P(\overline{A\cup B}) = 0$
$P(A\cup B) = 1 - P(\overline{A\cup B}) = 1 - 0 = 1$
We know,
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$1 = 0.5 + 0.6 - P(A\cap B)$
$1 = 1.1 - P(A\cap B)$
$P(A\cap B) = 0.1$
Consider,
$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0.1}{0.6} = \frac{1}{6}$

S71. Ans.(a)

S72. Ans.(d)

S73. Ans.(d)

Sol. Using Var x $= \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$

S74. Ans.(a)

Sol. First of all consider the radius of circle as r, than the points closer to center than boundary will lie within the radius of π/2. So the favourable outcome would be the points inside the area of circle with radius r/2. Whereas the total possible outcomes could be all the points inside the area of circle with radius r.

Therefore the probability is

$$= \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$$

S75. Ans.(b)

S76. Ans.(a)

S77. Ans.(d)

Sol. For f to be continuous $\lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x) = \lim_{x\to 0^+} f(x)$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sin(-x) = \lim_{h\to 0} \sin(-(0-h))$

$= \lim_{h\to 0} \sin h$

$= 0.$

$\lim_{x\to 0} f(x) = \lim_{x\to 0^-} f(x)$

$\lim_{x\to 0} k = 0$

$\Rightarrow$ k = 0.

S78. Ans.(b)

Sol. Given xy $= c^2$

$\Rightarrow$ y $= \frac{c^2}{x}$ ____(1)

Now,

$z = a^2x + b^2y$

$z = a^2x + b^2\frac{c^2}{x}[from\ (1)]$

Differentiate w.r.t x

$\frac{dz}{dx} = a^2 - \frac{b^2c^2}{x^2}$ _______(2)

Put $\frac{dz}{dx} = 0$

$a^2 - \frac{b^2c^2}{x^2} = 0$

$\frac{b^2c^2}{x^2} = a^2$

$\frac{b^2c^2}{a^2} = x^2$

$\pm\frac{bc}{a} = x$

Now again differentiate eq (2)

$\frac{d^2z}{dx^2} = \frac{2b^2c^2}{x^3}$

$at\ x = \frac{bc}{a}\frac{d^2z}{dx^2} = \frac{2a^3}{bc} > 0 \Rightarrow \text{minimum}$

at x $-\frac{bc}{a}\frac{d^2z}{dx^2} = -\frac{2\,a^3}{bc} < 0 \Rightarrow \text{maximum}$

Hence the minimum value is

$Z|_{x=\frac{bc}{a}} = \frac{b\,c}{a} + \frac{b^2c^2}{bc} \times a$

$= abc + abc$

$= 2\text{abc}$

S79. Ans.(a)

Sol. $\int e^{x\,Ln(a)}\,dx$

$= \int e^{Ln\,a^x} dx$

$= \int a^x dx$

$= \frac{a^x}{Ln(a)} + C$

S80. Ans.(d)

Sol. $2\int_0^{2\pi} c \sin x\ dx$

$= 2[-C \cos x]_0^{2\pi}$

$= 2[-c \cos \pi + c \cos 0]$

$= 2[c + c]$

$= 4\text{c}$

S81. Ans.(c)

Sol. $sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$sin\theta = \left(\sqrt{2} - 1\right)\cos\theta$ ______(1)

Now, $\cos\theta = \sin\theta$

$\cos\theta - \left[\left(\sqrt{2} - 1\right)cos\theta\right]\ [from\ (1)]$

$\cos\theta[1 - \sqrt{2} + 1]$

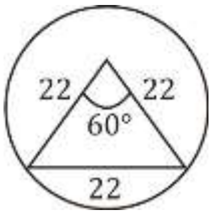
$\cos\theta\ [2 - \sqrt{2}]$

$\sqrt{2}\left(\sqrt{2} - 1\right)\cos\theta$

$\sqrt{2}\sin\theta$ [from (1)]

S82. Ans.(a)

Sol.

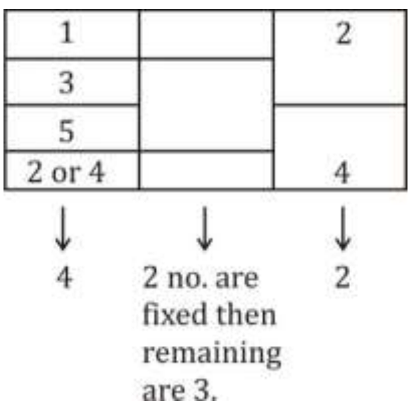


Length of minor arc of the chord $= \frac{60°}{360°} \times \frac{2\times22}{7} \times 22 = \frac{484}{21} cm$

S83. Ans.(c)

S84. Ans.(c)

Sol.



i.e. $4 \times 3 \times 2 = 24$

S85. Ans.(a)

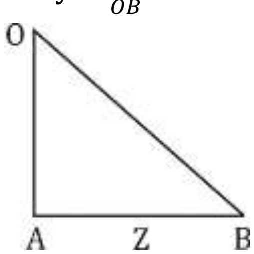
Sol. Let the length of the tower OD = h

In Δ DOA

$\tan x = \frac{h}{OA} \Rightarrow OA = \frac{h}{\tan x}$

In Δ DOB

$\tan y = \frac{h}{OB} \Rightarrow OB = \frac{h}{\tan y}$



In Δ OAB

$OA^2 + z^2 = OB^2$

$\frac{h^2}{\tan^2 x} + z^2 = \frac{h^2}{\tan^2 y}$

$z^2 = \frac{h^2}{\tan^2 y} - \frac{h^2}{\tan^2 x}$

$= h^2[\cot^2 y - \cot^2 x]$

S86. Ans.(d)
Sol. $\frac{4}{52} = \frac{1}{13}$ [as ace are 4 in 52 cards]

S87. Ans.(d)
Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.5 + 0.6 - 0.4 = 0.7$
$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$

S88. Ans.(a)
Sol.
$1 - \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4}$
$1 - \frac{3}{32}$
$= \frac{29}{32}$

S89. Ans.(c)
Sol. Probability $= \frac{15}{36} = \frac{5}{12}$

S90. Ans.(c)
Sol.
$x = \begin{Bmatrix} (1,H)(2,H)(3,H)(4,H)(5,H)(6,H) \\ (1,T)(2,T)(3,T)(4,T)(5,T)(6,T) \end{Bmatrix}$
Probability $= \frac{3}{12} = \frac{1}{4}$

S91. Ans.(c)
Sol. $f(r) = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$
$f(1) = 2$
$f(2) = 1$
$f(1) + f(2) = 2 + 1 = 3$

S92. Ans.(c)
Sol. Given $f(x) = 3^{1+x}$
$f(y) = 3^{1+y}$
$f(z) = 3^{1+z}$
$f(x)f(y)f(z) = 3^{1+x}.3^{1+y}.3^{1+z}$
$= 3^{1+(2+x+y+z)}$
$= f(2 + x + y + z)$

S93. Ans.(a)
Sol. Given $x^2 + 9|x| + 20 = 0$
$\Rightarrow x^2 + 9x + 20 = 0$ or $x^2 - 9x + 20 = 0$
By solving these equation we get x= 4, 5, –4, –5 but these values of x does not satisfy the given equation as |x| will always give positive value.
Hence the number of real roots of the given equation is zero.

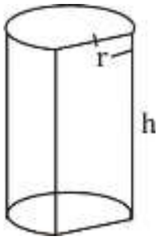
S94. Ans.(d)
Sol. $f(x) = \sin(\cos x)$
$f'(x) = \cos(\cos x)\frac{d}{dx}(\cos x)$
$= (-\sin x)\cos(\cos x)$

S95. Ans.(c)

S96. Ans.(a)
Sol.
$\frac{dy}{dx} = \cos(y - x) + 1$
Let y – x = t
$\frac{dy}{dx} - 1 = \frac{dt}{dx}$
$\Rightarrow \frac{dy}{dx} = 1 + \frac{dt}{dx}$
$\Rightarrow 1 + \frac{dt}{dx} = \cos t + 1$
$\Rightarrow \frac{dt}{dx} = \cos t$
⇒ sect dt = dx
Integrate both sides
$\Rightarrow \log(\sec t + \tan t) = x + a$
$\Rightarrow (\sec t + \tan t) = e^x . e^a$
$\Rightarrow \frac{e^x}{\sec t + \tan t} = e^{-a}$
$\Rightarrow \frac{e^x(\sec t - \tan t)}{(\sec t + \tan t)(\sec t - \tan t)} = e^{-a}$
$= \frac{e^x(\sec t - \tan t)}{\sec^2 t - \tan^2 t} = e^{-a}$
$= e^x(\sec(y - x) - \tan(y - x) = c$
Where c = e^{-a}

S97. Ans.(b)
Sol.
$\int_0^{\pi/2} |\sin x - \cos x|\, dx$
$\int_0^{\pi/4}(\cos x - \sin x)\, dx + \int_{\pi/2}^{\pi/2}(\sin x - \cos x) dx$
$[(\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$
$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - 1 - \left(\frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$
$= 2\sqrt{2} - 2$
$= 2(\sqrt{2} - 1)$

S98. Ans.(d)
Sol. y = a cos 2x + b sin 2x
$\frac{dy}{dx} = -2a\sin 2x + 2b\cos 2x$
$\frac{d^2y}{dx^2} = -4a\cos 2x - 4b\sin 2x$
$\frac{d^2y}{dx^2} = -4(a\cos 2x + b\sin 2x)$
$\frac{d^2y}{dx^2} + 4y = 0$

S99. Ans.(a)
Sol.


Surface area S $= \pi rh + \pi r^2 + 2rh$
$\frac{ds}{ar} = \pi h + 2\pi r + 2h$
Put $\frac{ds}{ar} = 0$
$\Rightarrow 2r = \frac{-(\pi h+2h)}{\pi}$
$\Rightarrow \frac{2r}{h} = -\frac{(\pi+2)}{\pi}$
$\Rightarrow \frac{h}{2r} = -\frac{\pi}{\pi+2}$
Neglecting '–' sign as h and r can not be negative.
$\Rightarrow \frac{h}{2r} = \frac{\pi}{\pi+2}$

S100. Ans.(b)
Sol. $\int_0^{\pi/2} e^{\sin x} \cos x \, dx$
Let sin x = t
cos x dx = dt
at x = 0 t = 0
at x = $\frac{\pi}{2}$ t = 1
$\int_0^1 e^t \, dt$
$= [e^t]_0^1$
$= e^1 - e^0$
$= e^1 - 1$

S101. Ans.(d)
Sol. Let y = $\frac{x-2}{x+2}$
$x - 2 = yx + 2y$
$x - y\,x = 2y + 2$
$x(1-y) = 2y + 2$
$x = \frac{2y+2}{1-y}$
$= \frac{2(y+1)}{1-y}$

S102. Ans.(a)
Sol. $\int \ln(x^2)dx$
$\int 2 \ln x \, dx$
$\ln x \int 2 \, dx - \int \frac{d}{dx} lnx \ \int 2 \, dx$
$\ln x \; 2x - \int \frac{1}{x}. 2x$
$2x \ln x - 2x + c$

S103. Ans.(b)
Sol. Let (x, y) be any point on $y^2 = 8x$
Then the distance between (x, y) and (4, 2) is
$D^2 = (x-4)^2 + (y-2)^2$
$D^2 = \left(\frac{y^2}{8} - 4\right)^2 + (y-2)^2 \left[\because y^2 = 8x \Rightarrow x = \frac{y^2}{8}\right]$
$\frac{d(D^2)}{dy} = 2\left(\frac{y^2}{8} - 4\right)\left(\frac{2y}{8}\right) + 2(y-2)$
$= \frac{y^2}{16} - 2y + 2y - 4$
$Put\, \frac{d(D^2)}{dy} = 0$
$\Rightarrow y^2 = 64 \Rightarrow y = 4$
$\Rightarrow \text{x} = \frac{16}{8} = 2$
$\text{D} = \sqrt{4+4} = 2\sqrt{2}$

S104. Ans.(c)
Sol. Equation of circle is
$(x-a)^2 + y^2 = a^2$
$x^2 + a^2 - 2xa + y^2 = a^2$
$x^2 - 2xa + y^2 = 0$ ___________(1)
Diff. w.r.t x
$2x - 2a + 2y\, y^1 = 0$
x + yy' = a
Put this value is equation (1)
$x^2 - 2x(x + yy') + y^2 = 0$
$x^2 - 2x^2 - sxy\, y' + y^2 = 0$
$-x^2 - 2xy\, y' + y^2 = 0$
$x^2 - y^2 + 2xy\, y' = 0$

S105. Ans.(c)

S106. Ans.(b)
Sol. Centroid $= \left(\frac{2+5+2}{3}, \frac{-3-3-3}{3}, \frac{3-4-2}{3}\right)$
$= \left(\frac{9}{3}, \frac{-9}{3}, \frac{-3}{3}\right) = (3, -3, -1)$

S107. Ans.(c)
Sol. We know that the general equation of the sphere is
$x^2 + y^2 + z^2 + 2gx + 2hy + 2kz + d = 0$
According to Question,
$-g = 3$
$-h = -4$
$-k = 5$
$r = \sqrt{g^2 + h^2 + R^2 - d}$
$= \sqrt{9 + 16 + 25 - 1}$
$= \sqrt{49}$
$= 7$

S108. Ans.(a)
Sol. $(2x + y + 2z - 9) + \lambda(4x - 5y - 4z - 1) = 0$
Put $(3, 2, 1)$
$6 + 2 + 2 - 9 + \lambda(12 - 10 - 4 - 1) = 0$
$1 + (-3)\lambda = 0$
$\lambda = \frac{1}{3}$
So,
$2x + y + 2z - 9 + \frac{4}{3}x \frac{-5}{3}y - \frac{4}{3}z - \frac{1}{3} = 0$
$\frac{10}{3}x - \frac{2}{3}y + \frac{2}{3}z = \frac{28}{3}$
10x – 2y + 2z = 28

S109. Ans.(a)
Sol. Given,
$4x - 2y + 4z + 9 = 0 \Rightarrow 8x - 4y + 8z + 18 = 0$
$and\ 8x - 4y + 8z + 21 = 0$
$d = \left|\frac{+18-21}{\sqrt{64+16+64}}\right|$
$= \left|\frac{-3}{\sqrt{144}}\right|$
$= \left|\frac{-3}{12}\right|$
$= \left|\frac{-1}{4}\right| = \frac{1}{4}$

S110. Ans.(d)

S111. Ans.(b)
Sol. $(\vec{b} - \vec{a}).(3\vec{a} + \vec{b})$
$\left[(2\hat{\imath} + \hat{\jmath} - 3\hat{k}) - (\hat{\imath} - 2\hat{\jmath} + 5\hat{k})\right].\left[3\hat{\imath} - 6\hat{\jmath} + 15\hat{k} + 2\hat{\imath} + \hat{\jmath} - 3\hat{k}\right]$
$= \left[\hat{\imath} + 3\hat{\jmath} - 8\hat{k}\right].\left[5\hat{\imath} - 5\hat{\jmath} + 12\hat{k}\right]$
$= 5 - 15 - 96$
$= -106$

S112. Ans.(d)
Sol. $AB = \sqrt{(2-3)^2 + (4+2)^2 + (-3-1)^2}$
$= \sqrt{(-1)^2 + (6)^2 + (-4)^2}$
$= \sqrt{1 + 36 + 16}$
$= \sqrt{53}$

S113. Ans.(a)
Sol. $\overrightarrow{AB}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BA} + \overrightarrow{CA}.\overrightarrow{CB}$
$= AB.AC\cos\theta + 0 + BC\ AC\sin\theta$
= AB. AB + BC. BC.
$AB^2 + BC^2$
$= AC^2$
$= P^2$

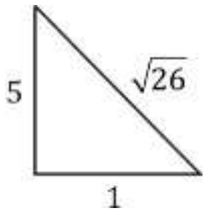
S114. Ans.(b)

Sol. $\cos\theta = \left|\frac{a.b}{|a||b|}\right|$

$= \left|\frac{8-18-3}{\sqrt{4+36+9}\,\sqrt{16+9+1}}\right|$

$= |\frac{-7}{7\sqrt{26}}$

$= \frac{1}{\sqrt{26}}$



$\Rightarrow \sin\theta = \frac{5}{\sqrt{26}}$

S115. Ans.(d)

Sol. For perpendicular

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$3\times(-2) + 4\times\lambda + (-1)\times 10 = 0$

$-6 + 4\lambda - 10 = 0$

$4\lambda = 16$

$\lambda = 4$

S116. Ans.(a)

Sol. $\frac{d}{dx}\sec^2(\tan^{-1}x)$

$= 2\sec(\tan^{-1}x).\sec(\tan^{-1}x)\tan(\tan^{-1}x).\frac{1}{1+x^2}$

$= 2\sec^2(\tan^{-1}x).\frac{x}{1+x^2}$

$= 2[1+\tan^2(\tan^{-1}x)].\frac{x}{1+x^2}$

$= 2(1+x^2).\frac{x}{1+x^2}$

$= 2x$

S117. Ans.(d)

Sol. $f(x) = \log_{10}(1+x)$

$4f(4) + 5f(1) - \log_{10}2$

$4\frac{\log 5}{\log 10} + \frac{5\log 2}{\log 10} - \frac{\log 2}{\log 10}$

$4\frac{\log 5}{\log 10} + 4\frac{\log 2}{\log 10}$

$4\left[\frac{\log 5+\log 2}{\log 10}\right] = 4\frac{\log 10}{\log 10} = 4$

S118. Ans.(b)

Sol. $f(x) = \ln\left(\sqrt{x^2+1} - x\right)$

$f(-x) = \ln\left(\sqrt{x^2+1} + x\right)$

$= \ln\left(\frac{(\sqrt{x^2+1}+x)(\sqrt{x^2+1}-x)}{(\sqrt{x^2+1-x})}\right)$

$= \ln\left(\frac{1}{\sqrt{x^2+1-x}}\right) = -\ln\left(\sqrt{x^2+1} - x\right)$

$= -f(x)$

S119. Ans.(d)

Sol. $f(x) = \log_x 10$

$= \frac{\log 10}{\log x}$

$f(x) = \frac{1}{\log x}$

S120. Ans.(c)

Sol. $\lim_{x\to 0} \frac{1-\cos^3 4x}{x^2} = \left(\frac{0}{0}\right)$ $form$

Apply L' Hospital rule

$\lim_{x\to 0} \frac{-3\cos^2 4x(-\sin 4x).4}{2x}$

$= \left(\frac{0}{0}\right)$ $form$

Again apply L' hospital rule.

$\lim_{x\to 0} + \frac{12[\cos^4 x \cos 4x.4 + \sin 4x \times 2\cos 4x(-\sin 4x).4]}{2}$

$\lim_{x\to 0} 6[4\cos^5 x - 8\sin^2 4x \cos 4\, x]$

$= 6\,[4] = 24$