MATHEMATICS FOR RRB ALP STAGE-II EXAM

Number System

1. L.C.M. and H.C.F. of Fractions

$$\text{L. C. M} = \frac{\text{L.C.M.of the numbers in numerators}}{\text{H.C.F.of in the number in denominator}}$$

$$\text{H. C. F} = \frac{\text{H.C.F.of the numbers in numerators}}{\text{L.C.M.of in the number in denominator}}$$

2. Product of two numbers

= L.C.M. of the numbers × H.C.F. of the numbers

3. To find the greatest number that will exactly divide x, y and z.

Required number = H.C.F. of x, y and z.

4. To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively.

Required number = H.C.F. of (x – a), (y – b) and (z – c).

5. To find the least number which is exactly divisible by x, y and z.

Required number = L.C.M. of x, y and z.

6. To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively. It is always observed that (x – a) = (y – b) = (z – c) = k (say)

∴ Required number = (L.C.M. of x, y and z) – k.

7. To find the least number which when divided by x, y and z leaves the same remainder r in each case.

Required number = (L.C.M. of x, y and z) + r

8. To find the greatest number that will divide x, y and z leaving the same remainder in each case.

(A) When the value of remainder r is given:

Required number = H.C.F. of (x – r), (y – r) and (z – r).

(B) When the value of remainder is not given:

Required number = H.C.F. of |(x – y)|, |(y – z)| and |(z – x)|

9. To find the n-digit greatest number which, when divided by x, y and z.

(A) leaves no remainder (i.e., exactly divisible)

Step 1: L.C.M. of x, y and z = L

Step 2: $\frac{\text{L)n−digit greatest number(}}{\text{Remainder=R}}$

Step 3: Required number = n-digit greatest number — R

(B) leaves remainder K in each case.

Required number = (n-digit greatest number — R) + K.

10. To find the n-digit smallest number which when divided by x, y and z.

(A) leaves no remainder (i.e., exactly divisible)

Step 1: L.C.M. of x, y and z = L

Step 2: $\frac{\text{L)n−digit smallest number(}}{\text{Remainder=R}}$

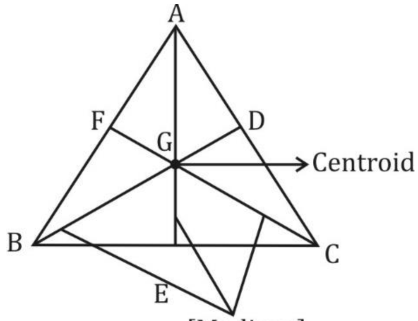
Step 3: Required number = n-digit smallest number + (L – R).

(B) leaves remainder K in each case.

Required number = n-digit smallest number + (L – R) + k.

Properties of Centers of Triangle

- **Centroid :**



[Medians]

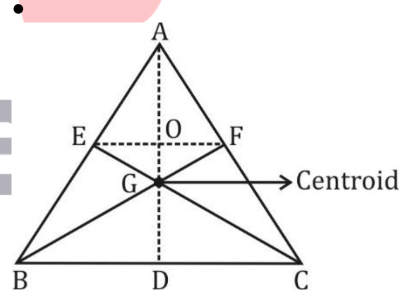
- Centroid divides median of a triangle in ratio 2 : 1

→ $\frac{AG}{GE} = \frac{2}{1}$

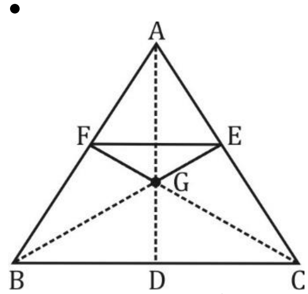
→ $\frac{AG}{AE} = \frac{2}{3}$

→ $\frac{GE}{AE} = \frac{1}{3}$

- area Δ ABE = $\frac{1}{2}$ area Δ ABC
- area Δ AGB = $\frac{1}{3}$ area Δ ABC
- area Δ AGF = $\frac{1}{6}$ area Δ ABC
-



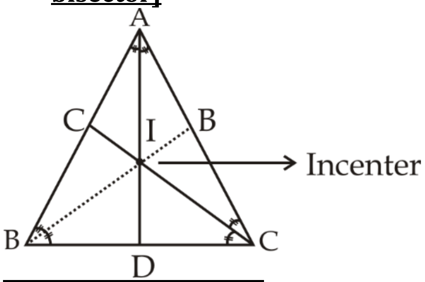
- AO = OD
- OG = $\frac{1}{3}$ AO
- $AB^2 + BC^2 = 2BD^2 + \frac{1}{2} AC^2$
- $AB^2 + AC^2 = 2AE^2 + \frac{1}{2} BC^2$
- $CA^2 + CB^2 = \frac{1}{2} AB^2 + 2 FC^2$
-



- Area Δ FGE = $\frac{1}{12}$ area Δ ABC

- 3 × (sum of side square) = 4x (sum of median square)
  3 × ($AB^2 + BC^2 + AC^2$) = 4x ($AE^2 + BD^2 + CF^2$)
- area Δ ABC = $\frac{4}{3}$ area Δ (Formed by taking AD, BF, CE, as sides of a triangle)

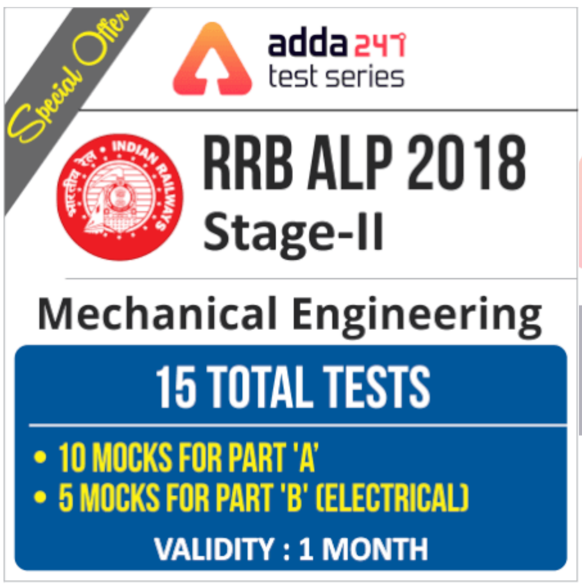
➢ **Incenter → [Intersecting Point of Internal angle bisector]**



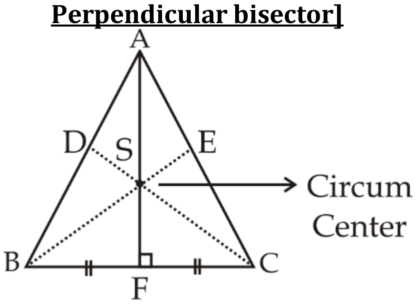
$\rightarrow \angle BIC = 90° + \frac{\angle A}{2}$

$\rightarrow$ IB = Inradius, r = $\frac{\text{area } \Delta ABC}{s}$

$\rightarrow$ AI : ID = b + c : a



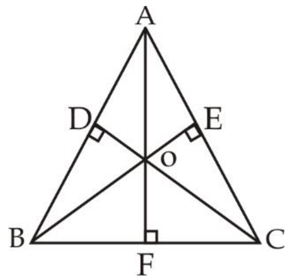
➢ **Circumcenter: → [Intersecting point of Perpendicular bisector]**



$\angle BSC = 2\angle A$

SB = circumradius, R = $\frac{abc}{4 \times \text{area } \Delta ABC}$

➢ **Orthocenter: → [Intersecting Point of Altitudes]**



$\angle BOC = 180 - \angle BAC$

➢ **Important Points: -**

(a) Orthocenter of right angled triangle ⇒ at right angled vertex

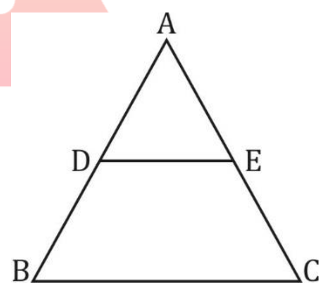
(b) Circumcenter of right angled triangle ⇒ Mid-point of Hypotenuse

(c) Distance b/w incenter & circumcenter of a triangle
$= \sqrt{R^2 - 2rR}$ $\begin{bmatrix} R = \text{circumradius} \\ r = \text{incente} \end{bmatrix}$

(d) In Equilateral triangle,
R = 2r,
Circum Radius : Inradius
2 : 1

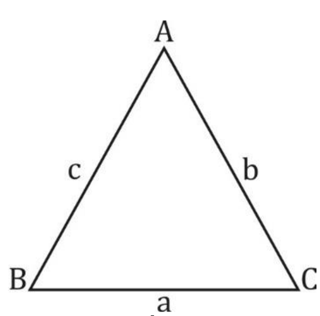
Properties of Triangle

➢



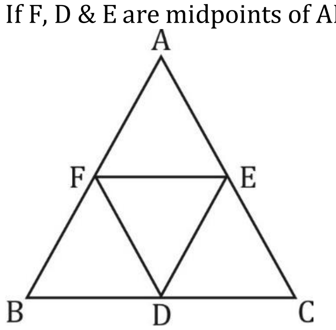
If DE||BC . Then,
$\frac{AD}{DB} = \frac{AE}{EC} = \frac{DE}{BC}$
$\frac{AD}{AB} = \frac{AE}{AC}$

➢



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ (R = Circumcenter)
a : b : c ≃ sin A : sin B : sin C

- If F, D & E are midpoints of AB, BC & AC



Then ,

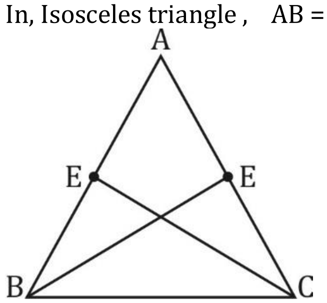
→ FE || BC & FE = $\frac{1}{2}$ BC

→ FD || AC & FD = $\frac{1}{2}$ AC

→ ED || AB & ED = $\frac{1}{2}$ AB

→ Area of ΔDFE = $\frac{1}{4}$ area ΔABC

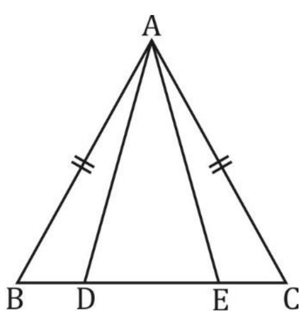
- In, Isosceles triangle , AB = AC



If E & F are mid points

Then, EC = BF

- In, Isosceles triangle, AB = AC & BE = CD



Then, AD = AE

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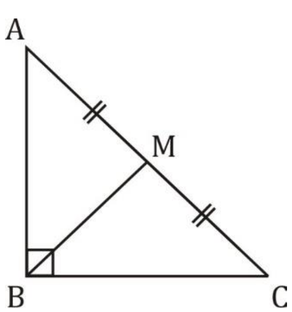

In a Right triangle, If M is the mid point of AC

AM = MC

then (तो)

BM = $\frac{1}{2}$AC

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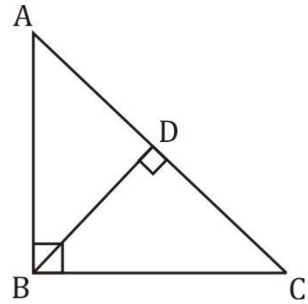


$\rightarrow BD = \frac{AB \times BC}{AC}$

$\rightarrow \frac{AD}{DC} = \frac{AB^2}{BC^2}$

$\rightarrow \frac{AB^2}{BD^2} = \frac{AC}{CD}$

$\rightarrow \frac{BC^2}{BD^2} = \frac{AC}{AD}$

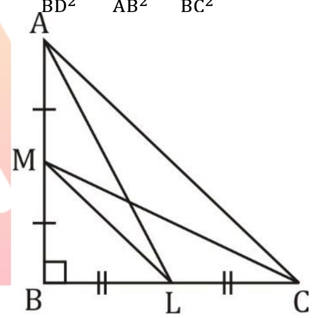
$\rightarrow \frac{AB^2}{BC^2} = \frac{AD}{CD}$

$\rightarrow BC^2 = AC \times DC$

$\rightarrow AB^2 = AC \times AD$

$\rightarrow BD^2 = AD \times DC$

$\rightarrow \frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$



(a) In a Right-angle Triangle If AL & CM are medians then $4\,(AL^2 + CM^2) = 5AC^2$

(b) If M & L are points anywhere on AB & BC then $AL^2 + CM^2 = AC^2 + ML^2$

TRIGONOMETRY

Important Results :-

- If $\tan\alpha.\tan\beta = 1$ then $\alpha + \beta = 90°$
- If $\sin\alpha.\sec\beta = 1$ then $\alpha + \beta = 90°$
- If $\cos\alpha.\text{cosec}\,\beta = 1$ then $\alpha + \beta = 90°$
- If $\cot\alpha.\cot\beta = 1$ then $\alpha + \beta = 90°$
- If $\sin\alpha.\text{cosec}\,\beta = 1$ then $\alpha + \beta = 180°$
- $\sin^2\theta + \cos^2\theta = 1$, $\sin^2\theta = 1 - \cos^2\theta$ & $\cos^2\theta = 1 - \sin^2\theta$
- $\sec^2\theta - \tan^2\theta = 1$, $\sec^2\theta = 1 + \tan^2\theta$ & $\tan^2\theta = \sec^2\theta - 1$
- $\text{cosec}^2\theta - \cot^2\theta = 1$, $\text{cosec}^2\theta = 1 + \cot^2\theta$ & $\cot^2\theta = \text{cosec}^2\theta - 1$
- If $\sin^2\alpha + \sin^2\beta = 2$ or $\sin\alpha + \sin\beta = 2$, then $\alpha = \beta = 90°$
- If $\sin^2 + \cos^2\beta = 2$ or $\sin\alpha + \cos\beta = 2$, then $\alpha = 90°$ & $\beta = 0°$
- if $\cos^2\alpha + \cos^2\beta = 2$ or $\cos\alpha + \cos\beta = 2$,then $\alpha = \beta = 0°$
- $\sin^2\alpha + \sin^2\beta = 0$ when $\alpha = \beta = 0$
- $\sin^2\alpha + \cos^2\beta = 0$ when $\alpha = 0$ & $\beta = 90°$
- $\cos^2\alpha + \cos^2\beta = 0$ when $\alpha = \beta = 90°$

Important Formulas

- $\sin(A + B) = \sin A\cos B + \cos A\sin B$
- $\sin(A - B) = \sin A\cos B - \cos A\sin B$
- $\cos(A + B) = \cos A\cos B - \sin A\sin B$
- $\cos(A - B) = \cos A\cos B + \sin A\sin B$
- $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$
- $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$
- $\sin 2A = 2\sin A\cos A = \frac{2\sin A}{\cos A}.\cos^2 A = \frac{2\tan A}{\sec^2 A} = \frac{2\tan A}{1+\tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A\left(1 - \frac{\sin^2 A}{\cos^2 A}\right) = \frac{1-\tan^2 A}{1+\tan^2 A}$
- $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
- $2\sin A\cos B = \sin(A + B) + \sin(A - B)$
- $2\cos A\sin B = \sin(A + B) - \sin(A - B)$
- $2\cos A\cos B = \cos(A + B) + \cos(A - B)$
- $2\sin A\sin B = \cos(A - B) - \cos(A + B)$
- $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$
- $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-D}{2}\right)$
- $\sin^2 A - \sin^2 B = \sin(A + B).\sin(A - B)$
- $\cos^2 A - \sin^2 B = \cos(A + B).\cos(A - B)$
- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Important Identity

- If $A + B + C = 180°$ or π (or ABC is a triangle), Then
$\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$
$\cot A.\cot B + \cot B.\cot C + \cot C.\cot A = 1$

- If $A + B + C = 90°$
$\cot A + \cot B + \cot C = \cot A.\cot B.\cot C$
$\tan A\tan B + \tan B\tan C + \tan C\tan A = 1$

- $\sin\theta.\sin(60° - \theta).\sin(60° + \theta) = \frac{1}{4}\sin 3\theta$
- $\cos\theta.\cos(60° - \theta).\cos(60° + \theta) = \frac{1}{4}\cos 3\theta$
- $\tan\theta.\tan(60° - \theta).\tan(60° + \theta) = \tan 3\theta$
- $\cot\theta.\cot(60° - \theta).\cot(60° + \theta) = \cot 3\theta$

Maximum & Minimum value of trigonometric Ratios :

	max^m	min^m
$\sin\theta$ or $\cos\theta$	1	–1
$\sin^2\theta$ or $\cos^2\theta$	1	0
$\sin^3\theta$ or $\cos^3\theta$	1	–1
$\tan\theta$ or $\cot\theta$	∞	$-\infty$
$\tan^2\theta$ or $\cot^2\theta$	∞	0
$\tan^3\theta$ or $\cot^3\theta$	∞	$-\infty$
$\sec\theta$ or $\text{cosec}\,\theta$	∞	$-\infty$
$\sec^2\theta$ or $\text{cosec}^2\theta$	∞	1
$\sec^3\theta$ or $\text{cosec}^3\theta$	∞	$-\infty$

- $a\sin\theta + b\cos\theta$

$\Rightarrow \text{max}^m \text{ value} = \sqrt{a^2 + b^2}$
$\Rightarrow \text{min}^m \text{ value} = -\sqrt{a^2 + b^2}$

- Maximum & minimum value of $\sin^n\theta.\cos^n\theta$

$\Rightarrow \text{max}^m\text{value} = \frac{1^n}{2^n} = \frac{1}{2^n}$ $[\because \text{max}^m\text{value of } \sin 2\theta = 1]$
$\Rightarrow \text{min}^m\text{value} = \frac{(-1)^n}{2^n} < \frac{-1}{2^n}$ n – odd
0 n – even

- $a\tan^2\theta + b\cot^2\theta$

$= \left(\sqrt{a}\tan\theta\right)^2 + \left(\sqrt{b}\cot\theta\right)^2 - 2\sqrt{a}\tan\theta.\sqrt{b}\cot\theta + 2\sqrt{a}\tan\theta\,.\sqrt{b}\cot\theta$
$= \left(\sqrt{a}\tan\theta - \sqrt{b}\cot\theta\right)^2 + 2\sqrt{a}\tan\theta\,.\sqrt{b}\cot\theta$
$= \left(\sqrt{a}\tan\theta - \sqrt{b}\cot\theta\right)^2 + 2\sqrt{ab}$
Min^m value will be $2\sqrt{ab}$ when $\sqrt{a}\tan\theta - \sqrt{b}\cot\theta = 0$
Min^m value = $2\sqrt{ab}$

- $a\tan^2\theta + \cot^2\theta$

$\Rightarrow$ **minimum value** = $2\sqrt{ab}$

- $a\sin^2\theta + b\,\text{cosec}^2\theta$

minimum value = $2\sqrt{ab}$ **when** $b \le a$, = a + b **when** $b \ge a$

- $a\cos^2\theta + b\sec^2\theta$

$\Rightarrow$ **minimum value** = $2\sqrt{ab}$ **when** $b \le a$
= a + b **when** $b \ge a$

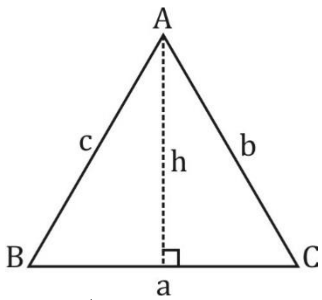
- $a\sec^2\theta + b\,\text{cosec}^2\theta$

Minimum value = $\left(\sqrt{a} + \sqrt{b}\right)^2$

Mensuration

Formulae of Areas of Different Triangles

- **Scalene triangle: →**



Area = $\frac{1}{2}$ × base × height

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

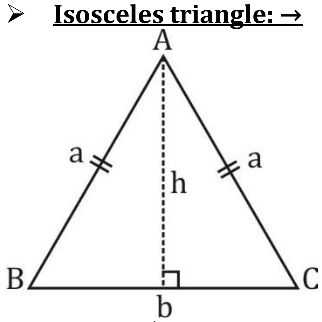
Where S = $\frac{a+b+c}{2}$

Area = $\frac{1}{2}$ × a × c sin B

= $\frac{1}{2}$ × a × b × sin C

= $\frac{1}{2}$ × b × c × sin A

- **Isosceles triangle: →**

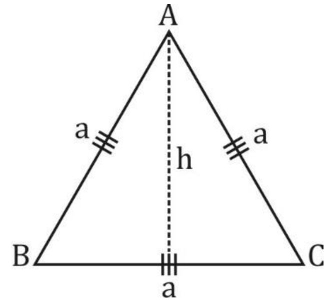


Height (h) = $\frac{1}{2}\sqrt{4a^2-b^2}$

Area = $\frac{1}{2}$ × base × height

Area = $\frac{1}{2}b\sqrt{4a^2-b^2}$

- **Equilateral triangle: →**

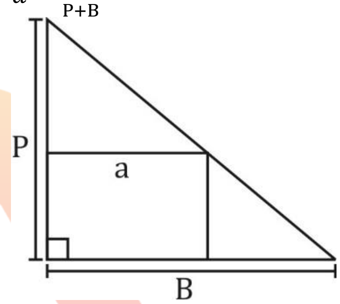


Area = $\frac{\sqrt{3}}{4}a^2$

h = $\frac{\sqrt{3}}{2}a$

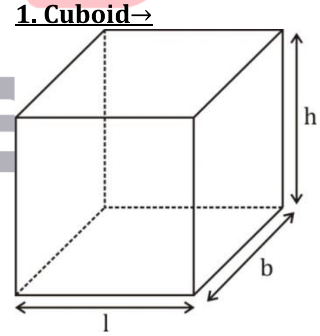
- **Right angled triangle: →**
  Side of the maximum size square inscribed in a right angle Δ =

$a = \frac{P \times b}{P+B}$



Formulae of Surface Area & Volume of Different 3D Figures

1. Cuboid→



- Volume of cuboid = l × b × h
- Lateral surface Area = Perimeter of Base × Height Base = 2(l + b) × h
- Total surface area = Lateral surface Area + 2 × Area of base = 2 (lh + bh + lb)
- Diagonal = $\sqrt{l^2+b^2+h^2}$
- V = $\sqrt{A_1 \times A_2 \times A_3}$

$A_1 \Longrightarrow$ Area of base or top = lb

$A_2 \Longrightarrow$ Area of one side face = bh

$A_3 \Longrightarrow$ Area of another side face = hl

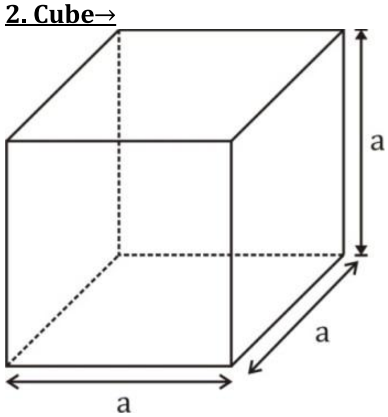
- To find the total surface area of a cuboid if the sum of all three sides and diagonals are given.

Total surface area = (sum of all three side)^2 – (Diagonal)^2

- For painting the surface area of a box or to know how much tin sheet is required, we will use, Total surface area.

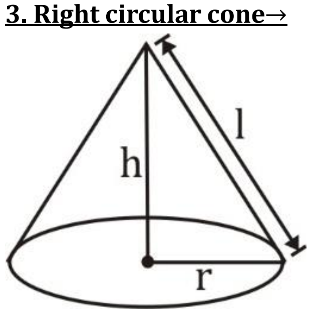
- To find the length of the longest pole to be placed is a room, we will calculate diagonal i.e. $\sqrt{l^2 + b^2 + h^2}$

2. Cube→



- Volume = $(\text{side})^3 = a^3$
- Lateral surface area = $4a^2$
- Total surface area = $6a^2$
- Diagonal of the cube = $\sqrt{3}\,a$
- Face diagonal of the cube = $\sqrt{2}a$
- Volume of cube = $\left(\sqrt{\frac{total\ surface\ area}{6}}\right)^3$
- In Radius of cube = $\frac{a}{2}$
- Circumradius of cube = $\frac{\sqrt{3}}{2}$ a

3. Right circular cone→



- Slant height, l = $\sqrt{r^2 + h^2}$
- Volume = $\frac{1}{3}$ × area of base × height = $\frac{1}{3}\ \pi r^2 h$
- Curved surface area = $\frac{1}{2}$ (Perimeter of base) × slant height

$= \frac{1}{2}\ \times\ 2\pi r \times l = \pi r l = \pi r\sqrt{r^2 + h^2}$

- Total surface area = C.S.A + Area of base

$= \pi r l + \pi r^2 = \pi r(l + r)$

- If cone is formed by sector of a circle, then.

(a) Slant height = radius of circle

(b) circumference of base of cone = length of arc of sector

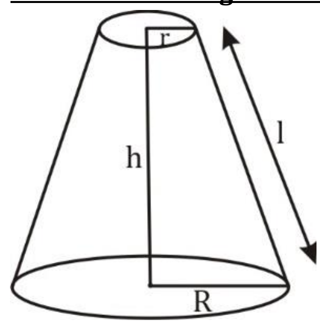
- Radius of maximum size sphere in a cone

$$= \frac{h\ \times r}{l\ +\ r} \qquad \begin{bmatrix} r\ \rightarrow\ radius\ of\ cone \\ l\ \rightarrow slant\ height\ of\ cone \\ h\ \rightarrow height\ of\ cone \end{bmatrix}$$

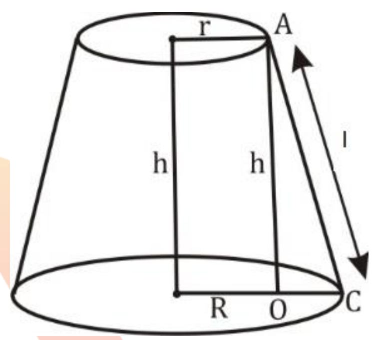
- If cone is cut parallel to its base and ratio of heights, radius or slant height of both parts is given as → x : y.

Then Ratio of their volume = $x^3\ :\ y^3$

4. Frustum of a right Circular cone →



- Slant height



AC = l, AB = h, BC = R – r

Applying Pythagorean theorem in ΔABC

L = $\sqrt{h^2 + (R - r^2)}$

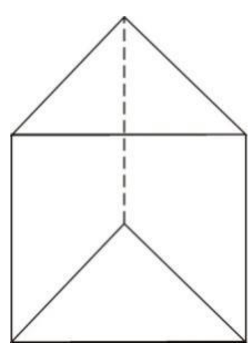
- volume of frustrum = $\frac{1}{3}\pi(R^2 + r^2 + Rr)h$
- Curved surface area = $\pi(R + r)l$
- Total surface area, T.S.A = $\pi(R + r)l + \pi(R^2 + r^2)$

5. Prism →

- A prism is a solid object with:

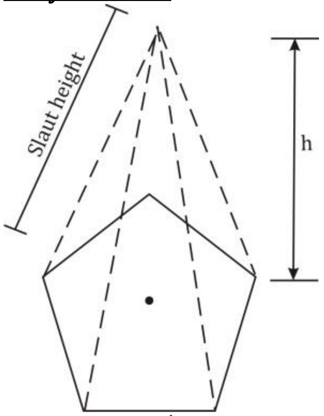
(a) Identical Ends

(b) Flat faces



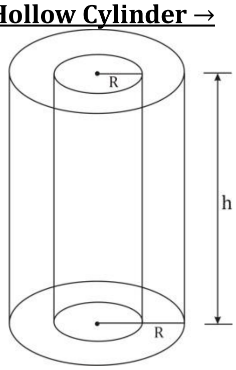
- Volume of Prism = Area of base × height
- Lateral surface area of prism = perimeter of base × height
- Total surface area of = Perimeter of base × height + 2 × area of base

6. Pyramids→



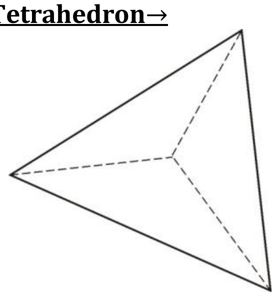
- Volume = $\frac{1}{3}(area\ of\ base) \times height$
- Curved surface area = $\frac{1}{2} \times (\text{perimeter of base}) \times \text{slant height}$
- Total surface area = curved surface area + area of the base
- Whenever in a question, If we want to find Slant height or height, then we will use inradius of the base not the Radius or side of the base.

7. Hollow Cylinder →



- Volume = $\pi(R^2 - r^2)h$
- Curved Surface Area = 2π(R+r)h
- Total surface area = 2π (R+r)h + $2\pi(R^2 - r^2)$

8. Tetrahedron→



- Height = $\sqrt{\frac{2}{3}}a$
- Volume = $\frac{\sqrt{2}}{12}a^3$
- Lateral surface area = $\frac{3\sqrt{3}}{4}a^2$
- Total surface area = $\sqrt{3}a^2$
- Slant height = $\frac{\sqrt{3}}{2}$a
- Slant Edge = a

9. Swimming Pool:

- Volume of swimming Pool = $\frac{1}{2}$ [Sum of depth of both sides] × length × Breadth

Simple Interest

1. If a certain sum in T years at R% per annum amounts to Rs. A, then the sum will be

$P = \frac{100 \times A}{100 + R \times T}$

2. The annual payment that will discharge a debt of Rs. A due in T years at R% per annum is

Annual payment = Rs. $\left(\frac{100\text{ A}}{100\text{T} + \frac{\text{RT(T}-1)}{2}}\right)$

3. If a certain sum is invested in n types of investments in such a manner that equal amount is obtained on each investment where interest rates are R_1, R_2, R_3 ……, R_n, respectively and time periods are T_1, T_2, T_3, ……, T_n, respectively, then the ratio in which the amounts are invested is

$\frac{1}{100+R_1T_1} : \frac{1}{100+R_2T_2} : \frac{1}{100+R_3T_3} : \dots \frac{1}{100+R_nT_n}$.

4. If a certain sum of money becomes n times itself in T years at simple interest, then the rate of interest per annum is

$R = \frac{100(n-1)}{T}\%$

5. If a certain sum of money becomes n times itself at R% per annum simple interest in T years, then

$T = \left(\frac{n-1}{R}\right) \times 100$ years

6. If a certain sum of money becomes n times itself in T years at a simple interest, then the time T in which it will become m times itself is given by

$T' = \left(\frac{m-1}{n-1}\right) \times T$ years

7. Effect of change of P, R and T on simple interest is given by the following formula:

Change in Simple Interest= $\frac{\text{Product of fixed parameter}}{100} \times$ [difference of product of variable parameters]

For example, if rate (R) changes from R_1 to R_2 and P, T are fixed, then

Change in SI = $\frac{PT}{100} \times (R_1 - R_2)$

Similarly, if principal (P) changes from P_1 to P_2 and R, T are fixed, then change in SI

$= \frac{RT}{100} \times (P_1 - P_2)$

Also, if rate (R) change from R_1 to R_2 and time (T) changes from T_1 to T_2 but principal (P) is fixed, then change in

SI = $\frac{P}{100} \times (R_1T_1 - R_2T_2)$

8. If a certain sum of money P lent out at SI amounts to A_1 in T_1 years and to A_2 in T_2 years, then

$P = \frac{A_1T_2 - A_2T_1}{T_2 - T_1}$ and $R = \frac{A_1 - A_2}{A_1T_2 - A_2T_1} \times 100\%$

9. If a certain sum of money P lent out for a certain time T amounts to A_1 at R_1 % per annum and to A_2 at R_2 % per annum, then

$P = \frac{A_2R_1 - A_1R_2}{R_1 - R_2}$ and $T = \frac{A_1 - A_2}{A_2R_1 - A_1R_2} \times 100$ years

10. If an amount P_1 lent at simple interest rate of R_1 % per annum and another amount P_2 at simple interest rate of R_2 % per annum, then the rate of interest for the whole sum is

$R = \left(\frac{P_1R_1 + P_2R_2}{P_1 + P_2}\right)$

Compound Interest

1. (a) The amount A due after t years, when a principal P is given on compound interest at the rate R% per annum is given by

$A = P\left(1 + \frac{R}{100}\right)^t$

(b) Compound interest (CI) = A – P = $P\left[\left(1 + \frac{R}{100}\right)^t - 1\right]$

(c) Rate of interest (R) = $\left[\left(\frac{A}{P}\right)^{\frac{1}{t}} - 1\right]$% p.a.

Note: Simple interest and compound interest for 1 year at a given rate of interest per annum are always equal.

2. If the interest is compounded half-yearly, then

(a) Amount (A) = $P\left(1 + \frac{R}{100 \times 2}\right)^{2t}$

(b) Compound interest (CI) $= P\left[\left(1 + \frac{R}{100 \times 2}\right)^{2t} - 1\right]$

(c) Rate (R) $= 2 \times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t} \times 2} - 1\right]$% p.a.

3. If the interest is compounded quarterly, then

(a) Amount (A) = $P\left(1 + \frac{P}{100 \times 4}\right)^{4t}$

(b) Compound interest (CI) = $P\left[\left(1 + \frac{R}{100 \times 4}\right)^{4t} - 1\right]$

(c) Rate (R) $= 4 \times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t} \times 4} - 1\right]\% \, p.a.$

In general, if the interest is compound n times a year, then

(a) Amount (A) = $P\left(1 + \frac{R}{100 \times n}\right)^{n \times t}$

(b) Compound interest (CI) = $P\left[\left(1 + \frac{R}{100 \times n}\right)^{n \times t} - 1\right]$

(c) Rate of interest (R) $= n \times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t} \times n} - 1\right]\% \, p.a.$

4. When the rates of interest are different for different years, say R_1, R_2, R_3 percent for first, second and third year, respectively, then

Amount $= P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)$

5. When the time is given in the form of fraction, say $2\frac{3}{4}$ years, then,

Amount $= P\left(1 + \frac{R}{100}\right)^2 \times \left(1 + \frac{\frac{3}{4}R}{100}\right)$

6. (a) The difference between the compound interest and the simple interest on a certain sum of money for 2 years at R% per annum is given by

CI – SI = $P\left(\frac{R}{100}\right)^2$ [in term of P and R]

and, CI – SI = $\frac{R \times SI}{2 \times 100}$ [in terms of SI and R]

(b) The difference between the compound interest and the simple interst on a certain sum of money for 2 years at R% per annum is given by

CI – SI = $P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right]$ [in terms of P and R]

And, CI – SI = $\frac{SI}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right]$ [in terms of SI and R]

7. If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

8. If a certain sum becomes n times in t years, then the rate of compound interest is given by

$$R = 100\left[(n)^{\frac{1}{t}} - 1\right]$$

9. If a certain sum of money at compound interest amounts to Rs. x in A years and to Rs. y in B years, then the rate of interest per annum is

$R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%$

10. If a loan of Rs. P at R% compound interest per annum is to be repaid in n equal yearly instalments, then the value of each instalment is given by

Rs. $\frac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2 + \cdots + \left(\frac{100}{100+R}\right)^n}$.

Number System

1. L.C.M. and H.C.F. of Fractions

$$\text{L.C.M} = \frac{\text{L.C.M.of the numbers in numerators}}{\text{H.C.F.of in the number in denominator}}$$

$$\text{H.C.F} = \frac{\text{H.C.F.of the numbers in numerators}}{\text{L.C.M.of in the number in denominator}}$$

2. Product of two numbers = L.C.M. of the numbers × H.C.F. of the numbers

3. To find the greatest number that will exactly divide x, y and z.
Required number = H.C.F. of x, y and z.

4. To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively.
Required number = H.C.F. of (x – a), (y – b) and (z – c).

5. To find the least number which is exactly divisible by x, y and z.
Required number = L.C.M. of x, y and z.

6. To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively. It is always observed that (x – a) = (y – b) = (z – c) = k (say)
∴ Required number = (L.C.M. of x, y and z) – k.

7. To find the least number which when divided by x, y and z leaves the same remainder r in each case.
Required number = (L.C.M. of x, y and z) + r

8. To find the greatest number that will divide x, y and z leaving the same remainder in each case.
(A) When the value of remainder r is given:
Required number = H.C.F. of (x – r), (y – r) and (z – r).
(B) When the value of remainder is not given:
Required number = H.C.F. of |(x – y)|, |(y – z)| and |(z – x)|

9. To find the n-digit greatest number which, when divided by x, y and z.
(A) leaves no remainder (i.e., exactly divisible)
Step 1: L.C.M. of x, y and z = L
Step 2: $\frac{\text{L)n−digit greatest number(}}{\text{Remainder=R}}$
Step 3: Required number = n-digit greatest number — R
(B) leaves remainder K in each case.
Required number = (n-digit greatest number — R) + K.

10. To find the n-digit smallest number which when divided by x, y and z.
(A) leaves no remainder (i.e., exactly divisible)

Step 1: L.C.M. of x, y and z = L
Step 2: $\frac{\text{L)n−digit smallest number(}}{\text{Remainder=R}}$
Step 3: Required number = n-digit smallest number + (L – R).

(B) leaves remainder K in each case.
Required number = n-digit smallest number + (L – R) + k

Profit & Loss

1. Gain percent
Gain % = $\frac{\text{Gain} \times 100}{\text{C.P.}}$
Loss percent
Loss% = $\frac{\text{Loss} \times 100}{\text{C.P.}}$

2. When the selling price and gain percent are given:
$\text{C.P.} = \left(\frac{100}{100+\text{Gain\%}}\right) \times \text{S.P.}$

3. When the cost and gain percent are given:
$\text{S.P} = \left(\frac{100+\text{Gain\%}}{100}\right) \times \text{C.P.}$

4. When the cost and loss percent are given:
$\text{S.P.} = \left(\frac{100-\text{Loss\%}}{100}\right) \times \text{C.P}$

5. When the selling price and loss percent are given.
$\text{C.P} = \left(\frac{100}{100-\text{Loss\%}}\right) \times \text{S.P}$

6. If a man buys x items for Rs. y and sells z items for Rs. w, then the gain or loss percent made by him is
$$\left(\frac{xw}{zy} - 1\right) \times 100\%$$

7. If the cost price of m articles is equal to the selling price of n articles, then % gain or loss
$= \left(\frac{m-n}{n}\right) \times 100$
[If m > n, it is % gain and if m < n, it is % loss]

8. If an article is sold at a price S.P.$_1$, then % gain or % loss is x and if it is sold at a price S.P.$_2$, then % gain or % loss is y. If the cost price of the article is C.P., then
$\frac{S.P_1}{100+x} = \frac{S.P_2}{100+y} = \frac{C.P.}{100} = \frac{S.P_1 - S.P_2}{x-y}$
Where x or y is –ve, if it indicates a loss, otherwise it is +ve.

9. If 'A' sells an article to 'B' at a gain/loss of m% and 'B' sells it to 'C' at a gain/loss of n% If 'C' pays Rs. z for it to 'B' then the cost price for 'A' is
$\left[\frac{100^2 z}{(100+m)(100+n)}\right]$
where m or n is –ve, it indicates a loss, otherwise it is +ve.

10. If 'A' sells an article to 'B' at a gain/loss of m% and 'B' sells it to 'C' at a gain/loss of n%, then the resultant profit/loss percent is given by
$\left(m + n + \frac{mn}{100}\right)$ (i)
where m or n is –ve, if it indicates a loss, otherwise it is +ve.

11. When two different articles are sold at the same selling price, getting gain/loss of x% on the first and gain/loss of y% on the second, then the overall% gain or % loss in the transaction is given by
$\left[\frac{100(x-y)+2xy}{(100+x)+(100+y)}\right]\%$
The above expression represents overall gain or loss accordingly as its sign is +ve or –ve.

12. When two different articles are sold at the same selling price getting a gain of x% on the first and loss of x% on the second, then the overall% loss in the transaction is given by
$$\left(\frac{x}{10}\right)^2 \%$$
Note that in such questions there is always a loss.

13. A merchant uses faulty measure and sells his goods at gain/loss of x%. The overall % gain/loss(g) is given by

$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{Faulty measure}}$

Note: If the merchant sells his goods at cost price, then x = 0.

14. A merchant uses y% less weight/length and sells his goods at gain/loss of x%. The overall % gain/loss is given by
$\left[\left(\frac{y+x}{100-y}\right) \times 100\right]\%$

15. A person buys two items for Rs. A and sells one at a loss of l% and other at a gain of g%. If each item was sold at the same price, then
(a) The cost price of the item sold at loss
$= \frac{A\,(100+\%\text{ gain})}{(100-\%\text{ loss})+(100+\%\text{ gain})}$
(b) The cost price of the item sold at gian
$= \frac{A\,(100-\%\text{ loss})}{(100-\%\text{ loss})+(100+\text{gain})}$

16. If two successive discounts on an article are m% and n%, respectively, then a single discount equivalent to the two successive discounts will be
$$\left(m + n - \frac{mn}{100}\right)\%$$

17. If three successive discounts on an article are l%, m% and n%, respectively, then a single discount equivalent to the three successive discounts will be
$\left[l + m + n - \frac{(lm+ln+mn)}{100} + \frac{lmn}{100^2}\right]\%$

18. A shopkeeper sells an item at Rs. z after giving a discount of d% on labelled price. Had he not given the discount, he would have earned a profit of p% on the cost price. The cost price of each item is given by
$\text{C. P.} = \left[\frac{100^2 z}{(100-d)(100+p)}\right]$

PIPE & CISTERN

1. If an inlet can completely fill the empty tank in X hrs, the part of the tank filled in 1 hr = $\frac{1}{X}$.

2. If an outlet can empty the full tank in Y hrs, the part of the tank emptied in 1 hr = $\frac{1}{Y}$.

3. If both inlet and outlet are open, net part of the tank filled in 1 hr = $\frac{1}{X} - \frac{1}{Y}$.

4. Two pipes A and B can fill (or empty) a cistern is given by $\left(\frac{XY}{X+Y}\right)$ hrs.

5. Three pipes A, B and C can fill a cistern in X, Y and Z hrs, respectively, while working alone. If all the three pipes are opened together, the time taken to fill the cistern is given by
$\left(\frac{X+Y+Z}{XY+YZ+ZX}\right)$ hrs.

6. Two pipes A and B can fill a cistern in X hrs. and Y hrs., respectively. There is also an outlet C. If all the three pipes are opened together, the tank is full in Z hrs. The time taken by C to empty the full tank is given by
$\left(\frac{XYZ}{XZ+YZ-XY}\right)$ hrs.

7. A tank takes X hrs to be filled by a pipe. But due to a leak, it is filled in Y hrs. The amount of time in which the leak can empty the full tank
$= \left(\frac{XY}{Y-X}\right)$ hrs.

8. A cistern has a leak which can empty it in X hrs. A pipe which admits Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hrs. The capacity of the cistern is
$\left(\frac{XYZ}{Z-X}\right)$ litres.

9. One fill pipe A is k times faster than the other fill pipe B.
(a) If B can fill a cistern in x hrs, then the time in which the cistern will be full, if both the fill pipes are opened together, is
$\left(\frac{x}{k+1}\right)$ hrs.
(b) If A can fill a cistern in y hrs, then the time in which the cistern will be full, if both the fill pipes are opened together, is
$\left(\frac{k}{k+1}\right) y$ hrs.

10. If one fill pipe A is k times faster and takes x minutes less time than the other fill pipe B, then
(a) the time taken to fill a cistern, if both the pipes are opened together is
$\left(\frac{kx}{(k-1)^2}\right)$ minutes.
(b) A will fill the cistern in
$\left(\frac{x}{k-1}\right)$ minutes.
(c) B will fill the cistern in
$\left(\frac{kx}{k-1}\right)$ minutes.

Time & Work

$\text{Time} = \frac{\text{Work done}}{\text{Efficiency}}$

- When work is same.

$\text{Time} \propto \frac{1}{\text{Efficiency}}$

- If A can do a piece of work is n days.

Then, per day working efficiency of A = $\frac{1}{n}$

- If working efficiency of A & B is → x : y

Then, time taken by A & B to finish the work is in the ratio → y : x

e.g. If A does three times faster work than 'B', then ratio of work done by A and B is 3 : 1.
Then
Ratio of time taken by A & B = 1 : 3

- If A can do a piece of work is x days and B can do a piece of work is 4 days, then both of them working together will do the same work in
  $\frac{xy}{x+y}$ days
- If A, B & C will working alone, can complete a work is x, y and z days, respectively, then they will together complete the work in
  $\frac{xyz}{xy+yz+zx}$
- Two persons A & B, working together, can complete a piece of work in x days. If A, working alone, can complete the work in y days, then B, working alone, will complete the work in
  $\Rightarrow \frac{xy}{y-x}$
- If A & B working together, can finish a piece of work is x days, B & C in 4 days, C & A in z days. Then , A + B + C working together will finish the job is
  $\Rightarrow \frac{2xyz}{xy+yz+zx}$
- If A can finish a work in x days and B is k times efficient than A, then the time taken by both A and B, working together to complete the work is
  $\frac{x}{1+k}$
- If A & B working together can finish a work in x days & B is k times efficient than A, then the time taken by
  A working Alone, will take ⇒ (k + 1) x
  B working Alone, will take ⇒ $\left(\frac{k+1}{k}\right)x$
- If A working Alone takes a days more than A & B & B working Alone takes b days more than A & B. Then , Number of days, taken by A & B working together to finish a job is = $\sqrt{ab}$

Time Speed & Distance

Distance = Time × Speed

- When Distance is constant
  Time ∝ $\frac{1}{\text{speed}}$
- When Time is constant
  Distance ∝ speed
- When speed in constant
  Distance ∝ Time
- Average speed = $\frac{\text{Total Distance}}{\text{Total Time Taken}}$
- When Distance is equal
  Average speed = $\frac{2xy}{x+y}$
  x, y → speeds

Relative Speed→

(a) When two bodies move in the same direction, Let speed of two bodies be S_A & S_B.
Relative speed = $S_A - S_B$

(b) When two bodies are moving in the opposite direction. Let the speed of two bodies be S_A & S_B.
Relative speed = $S_A + S_B$.

(c) When two bodies moving towards each other than time taken by them to meet.

D→ Distance between two bodies.
S_A, S_B→ Speed of two bodies.
T, time taken to meet other = $\frac{D}{S_A+S_B}$

(d) When two bodies are moving in opposite direction, time taken to meet.
D→ Distance between the two bodies.
S_A, S_B→ Speed of two bodies
T, time taken = $\frac{D}{S_A-S_B}$

(e) If two persons A & B, start at the same time from P and Q towards each other and after crossing they take T_1 & T_2 hrs in reaching Q & P

$\frac{S_A}{S_B} = \sqrt{\frac{T_B}{T_A}}$

Algebraic Identities

Cyclic Factors

(a) $\mathbf{a^2 (b-c) + b^2 (c-a) + c^2 (a-b)} = -(a-b)(b-c)(c-a)$
(b) $\mathbf{bc(b-c) + ca(c-a) + ab(a-b)} = -(a-b)(b-c)(c-a)$
(c) $\mathbf{a(b^2-c^2) + b(c^2-a^2) + c(a^2-b^2)} = (a-b)(b-c)(c-a)$

Formula for 3 & 4 terms

(d) $\mathbf{(a+b+c)^2} = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
(e) $\mathbf{(a+b+c+d)^2} = a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$
(f) $\mathbf{(a+b+c)^3} = a^3 + b^3 + c^3 + 3(b+c)(c+a)(a+b)$
(g) $\mathbf{x^4 + x^2y^2 + x^4} = (x^2 + xy + y^2)(x^2 - xy + y^2)$

Special Results

➢ **If, $x + \frac{1}{x} = a$**

Then,
(a) $x^2 + \frac{1}{x^2} = a^2 - 2$
(b) $x^3 + \frac{1}{x^3} = a^3 - 3a$
(c) $x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$
(d) $x^5 + \frac{1}{x^5} = a^5 - 5a^3 + 5a$
(e) $x^6 + \frac{1}{x^6} = a^6 - 6a^4 + 9a^2 - 2$

➢ **If $x^4 + \frac{1}{x^4} = a$, Then**

$x^2 + \frac{1}{x^2} = \sqrt{a+2}$
$x + \frac{1}{x} = \sqrt{\sqrt{a+2} + 2}$
$x - \frac{1}{x} = \sqrt{\sqrt{a+2} - 2}$

➢ **If $x - \frac{1}{x} = a$**

Then,

(a) $x^2 + \frac{1}{x^2} = a^2 + 2$

(b) $x^3 - \frac{1}{x^3} = a^3 + 3a$

(c) $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

(d) $x^5 - \frac{1}{x^5} = a^5 + 5a^3 + 5a$

(e) $x^6 + \frac{1}{x^6} = a^6 + 6a^4 + 9a^2 + 2$

➢ **If $x^4 - \frac{1}{x^4} = a$, then**

(a) $x^2 + \frac{1}{x^2} = \sqrt{a+2}$

(b) $x + \frac{1}{x} = \sqrt{b+2}\ \{b = \sqrt{a+2}\}\ \sqrt{\sqrt{a+2}+2}$

(c) $x - \frac{1}{x} = \sqrt{b-2}\ \{b = \sqrt{a+2}\}\ \sqrt{\sqrt{a+2}-2}$

➢ **If $x + \frac{1}{x} = 1$**

Then, $\mathbf{x^3 = -1}$

➢ **If $x + \frac{1}{x} = -1$**

Then, $x^3 = 1$

➢ **If $x + \frac{1}{x} = \sqrt{3}$**

Then, $\mathbf{x^3 + \frac{1}{x^3} = 0}$, And $\mathbf{x^6 = -1}$

➢ $\mathbf{x^4 + x^2y^2 + y^4} = (x^2 + xy + y^2)(x^2 - xy + y^2)$

➢ **Useful identity**

(a) If $\mathbf{x^2 + \frac{1}{x^2} = 1}$, then $x^6 = -1$

(b) If $\mathbf{x^2 + \frac{1}{x^2} = -1}$, then $x^6 = 1$

➢ **If ax +bx = m & bx – ay = n**

Then,

$\boxed{(a^2 + b^2)(x^2 + y^2) = m^2 + n^2}$

<u>Arithmetic & Geometric Progression</u>

<u>Arithmetic Progression</u>

➢ **Series of the form→**

a, a + d, a + 2d, a + 3d . . . .

is called Arithmetic Progression, when they increase or decrease by a common difference

e.g.→ 5, 12, 19, 26, 33

Common difference, d = 12 – 5 = 7

19 – 12 = 7

26 – 19 = 7

➢ $a_1, a_2, a_3 \ldots\ldots\ldots, T_n$

d = common difference

= 2nd term – Ist term

$= a_2 - a_1$

$\boxed{T_n = a + (n-1)d}$

a → 1st term

n → no. of terms

d → Common difference

➢ Sum of n terms in A.P

$S_n = \frac{n}{2}[2a + (n-1)d]$

Or

$S_n = \frac{n}{2}[\text{First term} + \text{last term}]$

➢ Total no. of term

$= \frac{\text{Last term} - \text{First term}}{\text{Common difference}} + 1$

Geometric Progression

➢ $a, ar, ar^2, ar^3 \ldots\ldots\ldots\ldots\ldots ar^{n-1}$

Common Ratio (r) = $\frac{\text{Second term}}{\text{First term}}$

n^{th} term $(T_n) = ar^{n-1}$

➢ Sum of G.P, Sn = $\frac{a(r^n - 1)}{r-1}$ where r > 1

Or

$S_n = \frac{a(1-r^n)}{1-r}$ where $r < 1$

➢ Sum of an Infinite Geometric Progression when r < 1

$S_\infty = \frac{a}{1-r}$

➢ The figure made by joining the mid-points of a square and the area of thus made square is half the actual square.

➢ The figure made by joining the mid-points of a right-angle triangle is right angle triangle and the area thus made triangle is 1/4th of the actual triangle.

➢ To find the sum of First n natural number.

S = 1 + 2 + 3 + 4 + ... n

$$\boxed{S = \frac{n(n+1)}{2}}$$

- To find the sum of the Squares of the 1st n natural numbers.

$S = 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2$

$$\boxed{S = \frac{n(n+1)(2n+1)}{6}}$$

- To find the sum of the cubes of the 1st n natural numbers.

$S = 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3$

$$\boxed{S = \left[\frac{n(n+1)}{2}\right]^2}$$

- To find the sum of first n odd natural numbers.

S = 1 + 3 + 5 + ... + (2n – 1)

$S = n^2$

- To find the sum of first n even natural numbers

S = 2 + 4 + 6 + ... + 2n

$\boxed{S = n(n+1)}$

Quadratic Equations

An algebraic expression of the form: $ax^2 + bx + c = 0$, where a ≠ 0, b, c ∈ R is called a quadratic equation.

Root of the Quadratic Equation:

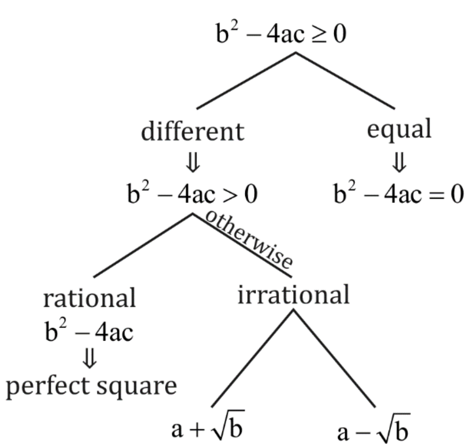
A root of the quadratic equation $ax^2 + bc + c = 0$ is a number α (real or complex) such that $a\alpha^2 + b\alpha + c = 0$ then $(x - \alpha)$ is factor of $ax^2 + bx + c$. The roots of the quadratic equation are

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Nature of Roots:

The value of x at which value of equation will be zero.

1. Roots are imaginary: $b^2 - 4ac \le 0$
2. Roots are real: $b^2 - 4ac \ge 0$



Sum & product of root:

Let there are two roots named α & β, then

$a = \frac{-b+\sqrt{b^2-4ac}}{2a}$ & $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

Sum of root: $\alpha + \beta = \frac{-b}{a}$

Product of root: $\alpha\beta = \frac{c}{a}$

Then, $ax^2 + bc + c = 0$ can be written as:

$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\Rightarrow x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$

$\boxed{x^2 - (\text{sum of root})x + \text{product of root} = 0}$

- If the roots α & β be reciprocal to each other, then a = c.
- If the two roots α & β be equal in magnitude and opposite in sign, then b = 0
- If a, b, c are rational number and $a + \sqrt{b}$ is one root of the quadratic equation, then the other root must be the conjugate $a - \sqrt{b}$ and vice-versa.

Condition for common Roots:

Let two quadratic equations–

$a_1x^2 + b_1x + c_1 = 0$ ... (i)

$a_2x^2 + b_2x + c_2 = 0$ ... (ii)

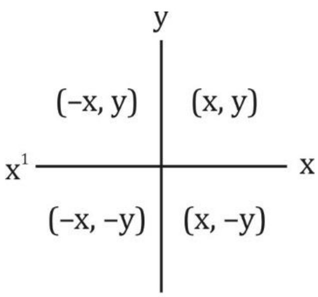
(A) If one root is common then,

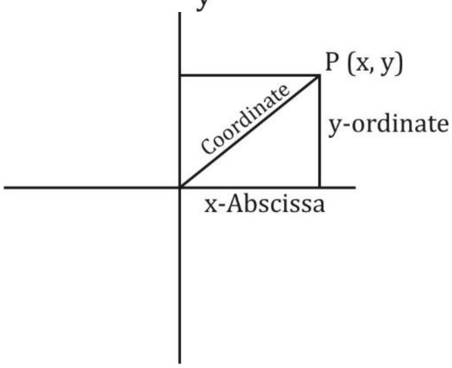
$\Rightarrow (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (c_1a_2 - c_2a_1)^2$

(B) If two roots are common then,

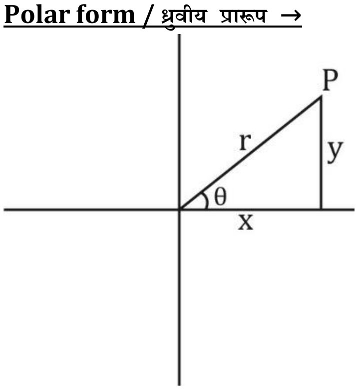
$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Coordinate Geometry





Polar form / ध्रुवीय प्रारूप →



$r = \sqrt{x^2 + y^2}$

$\sin\theta = \frac{y}{r} \rightarrow \boxed{y = r\sin\theta}$

$\cos\theta = \frac{x}{r} \rightarrow \boxed{x = r\cos\theta}$

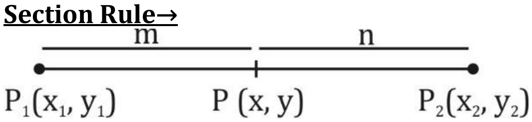
Coordinates of Points in Polar Form / ध्रुवीय प्रारूप में निर्देशांक बिंदु

$\rightarrow \boxed{(r\cos\theta, r\sin\theta)}$

Distance between two points / दो बिन्दुओं के बीच की दूरी →

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Section Rule→



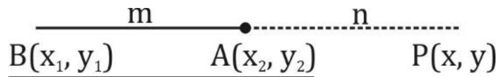
If, Point P(x, y) divides the line segment joining the points $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$ in the ratio m:n, internally.

$\boxed{x = \frac{mx_2+nx_1}{m+n},\ y = \frac{my_2+ny_1}{m+n}}$

(b) Coordinates of Mid-point / मध्य बिन्दुओं के निर्देशांक

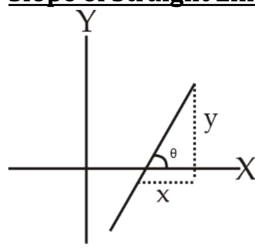
$\boxed{x = \frac{x_1+x_2}{2},\ y = \frac{y_2+y_1}{2}}$

(c) If division is External / यदि विभाजन बाह्य है:



$\boxed{x = \frac{mx_2-nx_1}{m-n},\ y = \frac{my_2-my_1}{m-n}}$

Slope of Straight Line / सरल रेखा की ढ़ाल



Slope (m) = tan θ = $\frac{y}{x}$

➢ When a straight-line passes through two points $P(x_1, y_1)$ & $Q(x_2, y_2)$ then slope of PQ

Slope (m) $= \frac{y_2-y_1}{x_2-x_1}$

Equations of straight line / सरल रेखा के समीकरण

I. If a point & slope is given, Point P (α, β) & Slope - m
Equation of line is, **$y - \beta = m(x - \alpha)$**

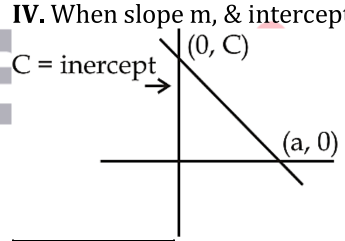
II. If two points $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$ are given
Slope, **m** $= \frac{y_2-y_1}{x_2-x_1}$

Equation of line is, **$y - y_1 = m(x - x_1)$**

III. When angle θ made with the axis & one point $P(x_1$ & $y_1)$ is given

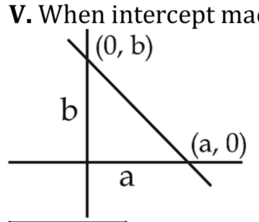
$\boxed{\begin{array}{c} m = \tan\theta \\ y - y_1 = m(x - x_1) \end{array}}$

IV. When slope m, & intercept made on y axis is given



$\boxed{y = mx + c}$, c ⇒ intercept made on axis

V. When intercept made on x-axis & y-axis both are given



$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$

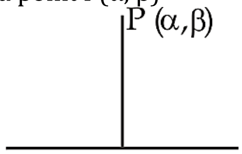
VI. General form of straight line, ax + by + c = 0
Slope (m) $= -\frac{a}{b}$

➢ Conditions of intersection of Two Straight Lines.

(a) If, **$m_1m_2 = -1$**, then **lines are perpendicular**

(b) If, **$m_1 = m_2$**, then **lines are parallel.**

- Two find the length of perpendicular on a straight line drawn from a point P(α, β)



- Gen. Eqn ax +by+c=0

Length of Perpendicular = $\frac{a\alpha+b\beta+c}{\sqrt{a^2+b^2}}$

- To Find Angle Between Two Straight Lines

$\boxed{\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|}$

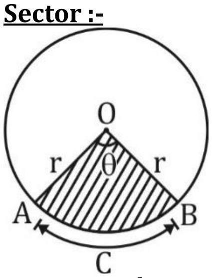
$m_1 \rightarrow$ slope of 1st line
$m_2 \rightarrow$ slope of 2nd line

- Distance Between Two Parallel Lines ax +by + c_1 = 0 And ax + by + c_2= 0

Is = $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

Circle

Sector :-



$\Rightarrow \theta = \frac{l}{r}$ $l \rightarrow$ length of arc AB, $r \rightarrow$ radius
↓
always in Radian

$I^C = \frac{180°}{\pi}$

$\pi^C = 180°$

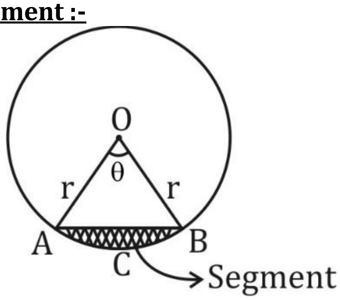
$I° = \frac{\pi^C}{180}$

$\Rightarrow$ Length of arc $= 2\pi r\frac{\theta}{360°}$

$\Rightarrow$ Area of sector OAB $= \pi r^2 \frac{\theta}{360°}$

$\Rightarrow$ Perimeter of sector $= \pi r\frac{\theta}{180°} + 2r$

Segment :-



$\rightarrow$ Area of segment = area of sector OACB – area of ΔOAB

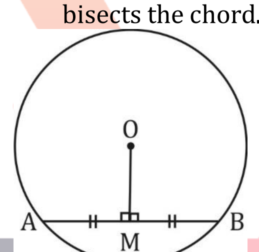
$= \pi r^2 \frac{\theta}{360°} - \frac{1}{2}r^2\sin\theta$

$\rightarrow$ Perimeter = length of arc ACB + Chord length AB

$= (2\pi r)\frac{\theta}{360°} + 2r\sin\left(\frac{\theta}{2}\right)$

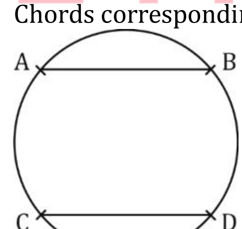
Important Properties Of Circle : -

- Perpendicular from the center of a circle to a chord bisects the chord.



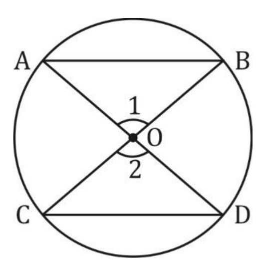
AM = MB

- Chords corresponding to equal arcs are equal.



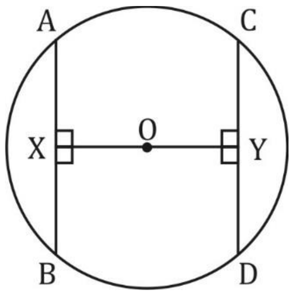
If $\widehat{AB} = \widehat{CD}$, then chord , AB = CD

- Equal Chords of Circle Subtends equal angles at the center.



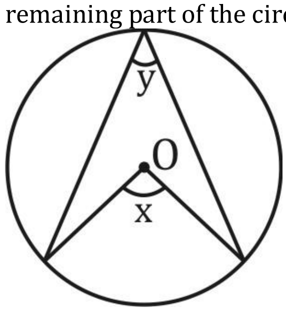
If AB = CD
then ∠1 = ∠2

➢ Equal chords of a circle are equidistance from the center.



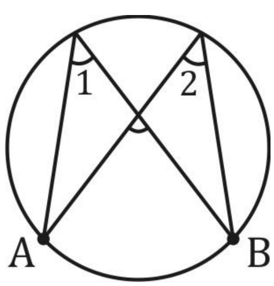
If AB = CD, Then OX = OY

➢ The angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circle



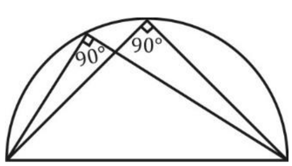
x = 2y

➢ Angle in same segment of a circle are equal.

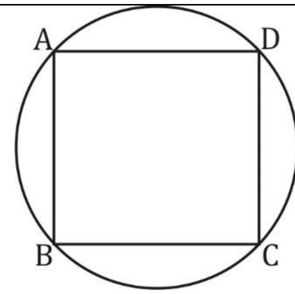


∠1 = ∠2

➢ Angle in a semicircle is always a right angle.

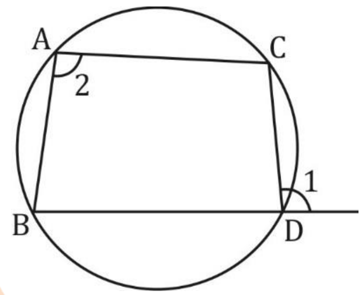


➢ If , ABCD is a cyclic quadrilateral



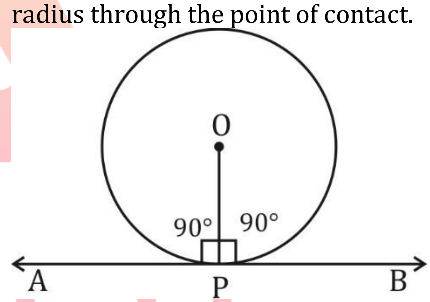
∠A + ∠C = 180°
∠B + ∠D = 180°

➢ ABCD is a cyclic quadrilateral.



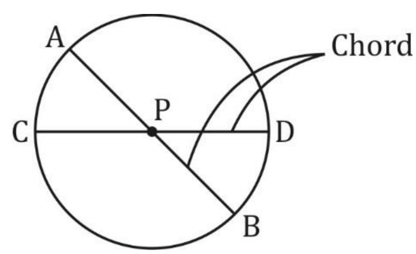
∠1 = ∠2

➢ A tangent at any point of circle is Perpendicular to the radius through the point of contact.

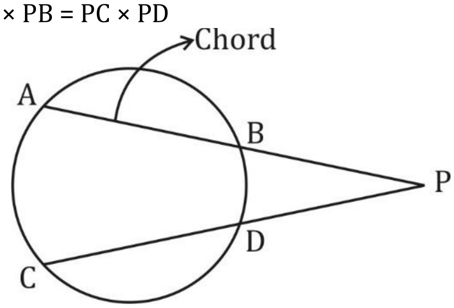


OP⊥AB

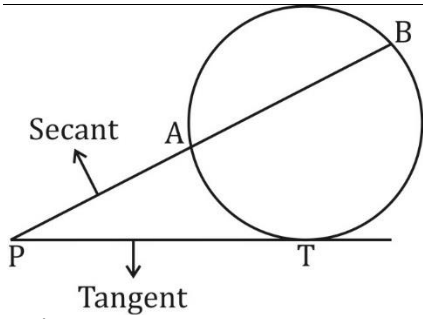
•



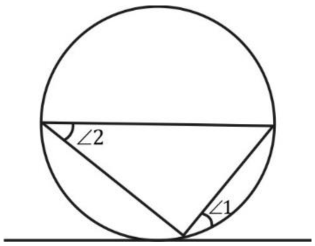
PA × PB = PC × PD



PA × PB = PC × PD

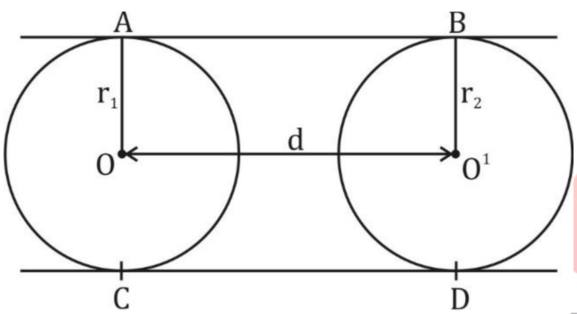


$PT^2 = PA \times PB$



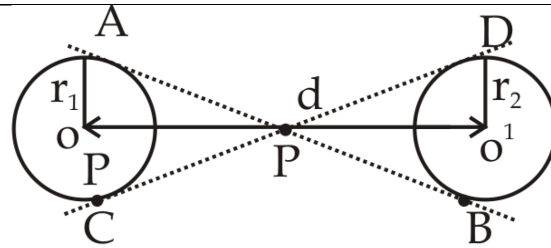
$\angle 1 = \angle 2$

➢ AB = CD = Direct Common tangent (उभयनिष्ठ अनुस्पर्शरेखा)



$AB = CD = \sqrt{d^2 - (r_1 - r_2)^2}$

➢ AB = CD Transverse Common Tangents (तिर्यक उभयनिष्ठ स्पशरिखा)



$AB = CD = \sqrt{d^2 - (r_1 + r_2)^2}$