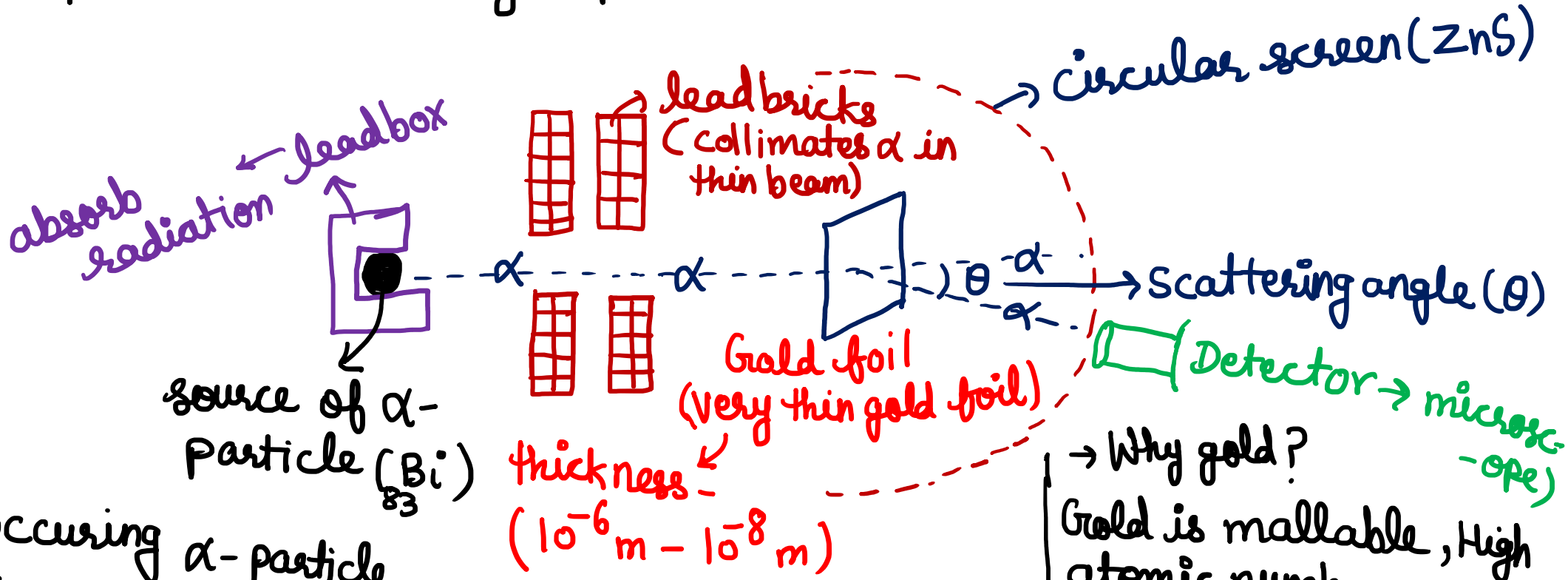
[Alpha - Particle Scattering Experiment]

In 1911, H. Geiger and Ernst Marsden (on suggestion of Ernst Rutherford) conducted α-particle scattering experiment -



→ Why gold?

Gold is malleable, High atomic number, gold foil can be made easily.

Naturally occuring α-particle (5 MeV - 7.7 MeV)

$\alpha \rightarrow He^{++} \rightarrow$ Helium Nucleus $(_{2n}^{2p}) \rightarrow$ 4 amu

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Observations –

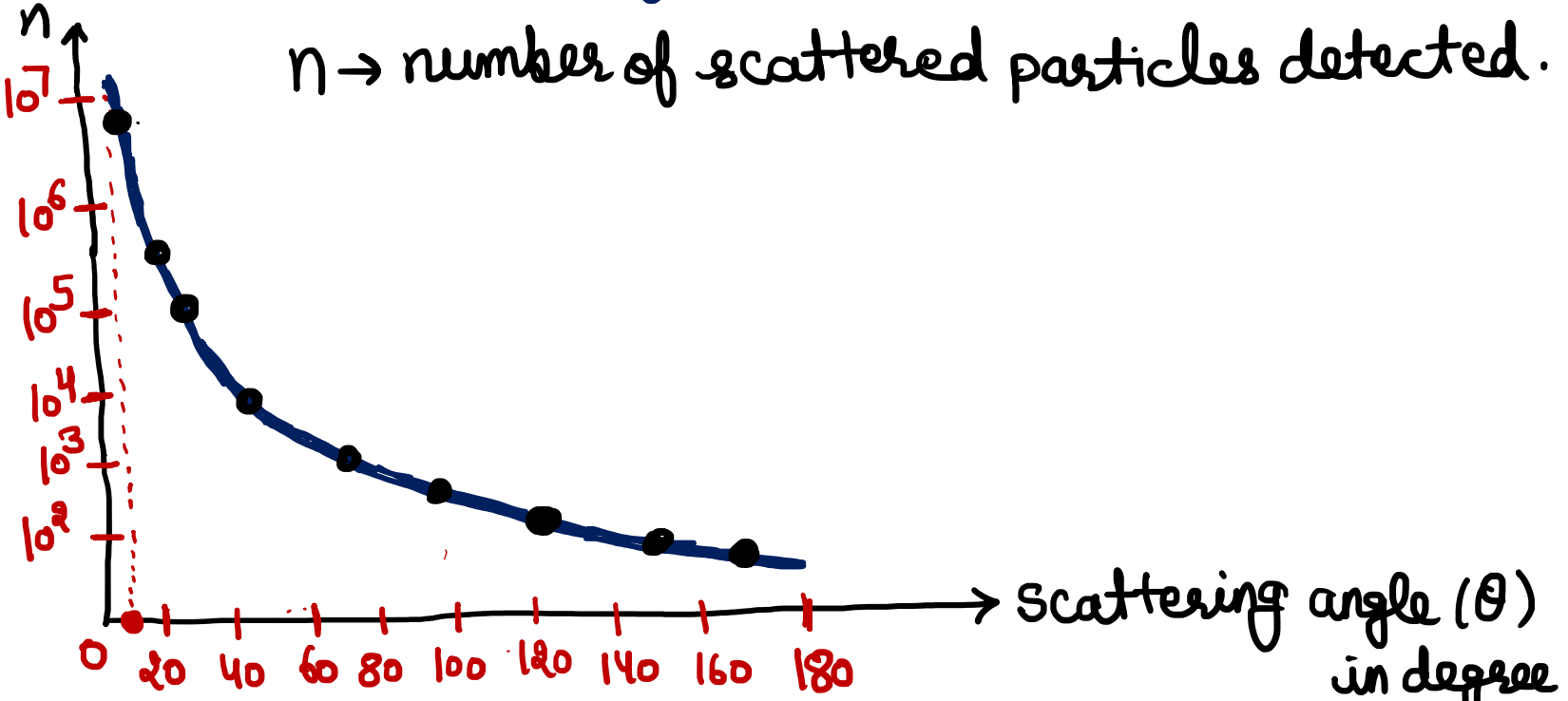
i) Most of the α- particles (99.86%) passes undeviated ($\theta = 0°$ or $\theta < 1°$)

ii) Only 0.14% α- particles deviated more than 1° ($\theta > 1°$)

iii) 1 in 8000 α- particle deviate more than 90° ($\theta > 90°$)

iv) 1 in 10,000,00 α- particle deviate by 180° ($\theta = 180°$ and return back)

In a sample of 10^7 α- particle sample, only 80-90 α-particle Scattered at 160°-170°.



n → number of scattered particles detected.

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Number of α-particle scattered at angle 'θ' –

Approx Formula,

$$N = \frac{KZ^2}{\sin^4\left(\frac{\theta}{2}\right)} \Rightarrow N \propto \frac{1}{\sin^4(\theta)}$$

where, N → number of α-particles scattered
Z → Atomic number of foil
θ → Scattering angle
K → constant for a given material (foil)

Q.1. If 1000 α-particles deflects at 60°, find number of α-particles that deflects at 90° for the same foil.

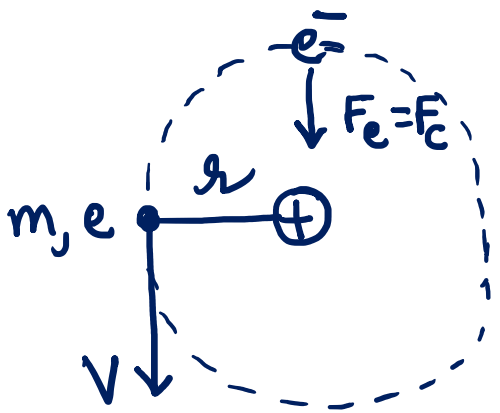
a) 100 b) 150 c) 200 d) 250

Sol.

$$\frac{N'}{N} = \frac{\frac{KZ^2}{\sin^4\left(\frac{90}{2}\right)}}{\frac{KZ^2}{\sin^4\left(\frac{60}{2}\right)}} = \frac{\sin^4\left(\frac{60}{2}\right)}{\sin^4\left(\frac{90}{2}\right)} = \frac{\sin^4 30^\circ}{\sin^4 45^\circ} = \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{\sqrt{2}}\right)^4} = \frac{1}{4} \Rightarrow N' = \frac{1}{4} \times 1000 = 250$$

Rutherford's Nuclear Model of Atom -

1. Since most α-particles passes undeviated, most part of atom is empty space (Atom is hollow from inside).
2. Some α-particles deflected by more than 1° (only 0.14%), this means there is a +ve charge inside atom as α is also +ve and this +ve charge is concentrated in an extremely small space inside atom called nucleus). Calculation show size of Nucleus is around 10^{-15} m whereas size of atom is around 10^{-10} m.
3. For foils of different elements, the number of α-particles deflected at angle θ are different. This suggests different elements (metal as he used silver, platinum etc) have different +ve charge in their nucleus.
4. The electrons are attracted by nucleus due to electrostatic force but are not pulled towards it as they are constantly revolving in circular paths, using this electrostatic force as centripetal force.



$$F_e = F_c \Rightarrow \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = \frac{mv^2}{r}$$

$$\text{or } \frac{1}{4\pi\epsilon_0}\frac{e(Ze)}{r^2} = \frac{mv^2}{r}$$

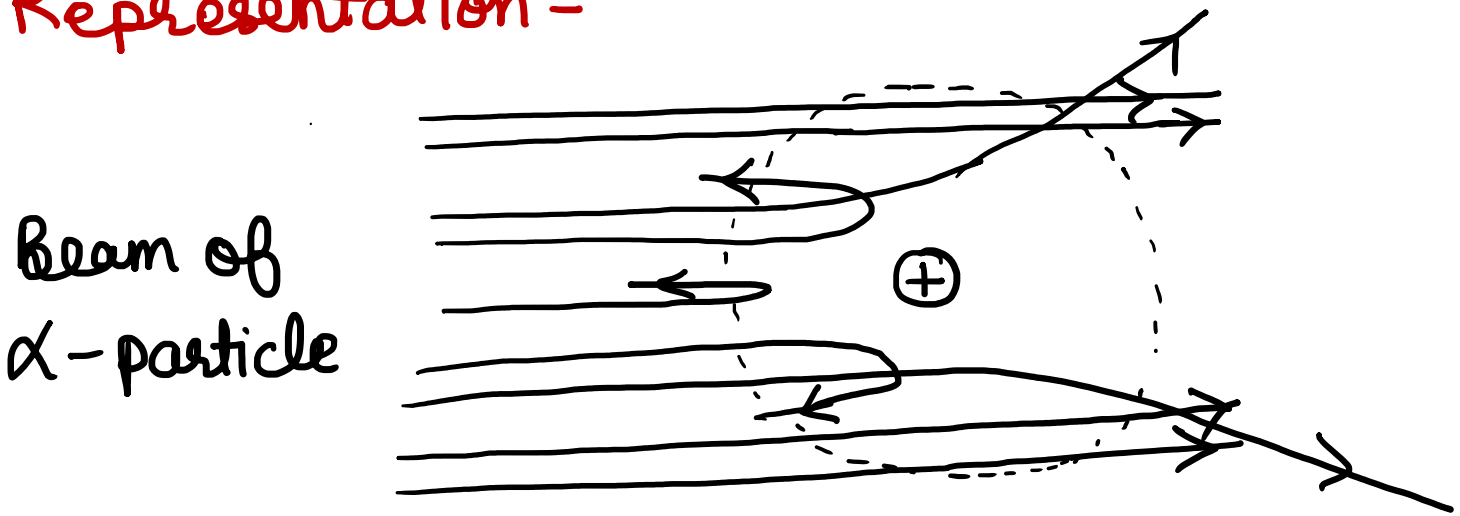
} Bohr's model explains.

where,

q_1 = Charge on e^- = e

q_2 = Charge on nucleus = Ze

* Pictorial Representation -



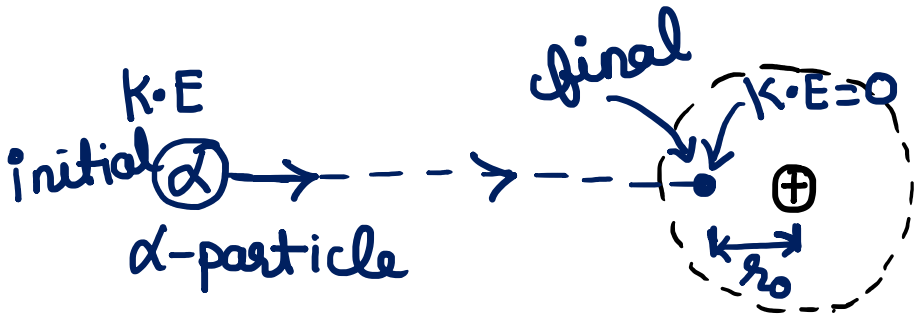
Nucleus size determination- distance of closest approach (r_0) -

Conservation of mechanical energy

$$U_i + K_i = U_f + K_f$$

$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\infty} + KE = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_0} + 0$$

$$K.E = \frac{2e\,(Ze)}{4\pi\epsilon_0 r_0} \Rightarrow \boxed{r_0 = \frac{2Ze^2}{4\pi\epsilon_0 (KE)}}$$

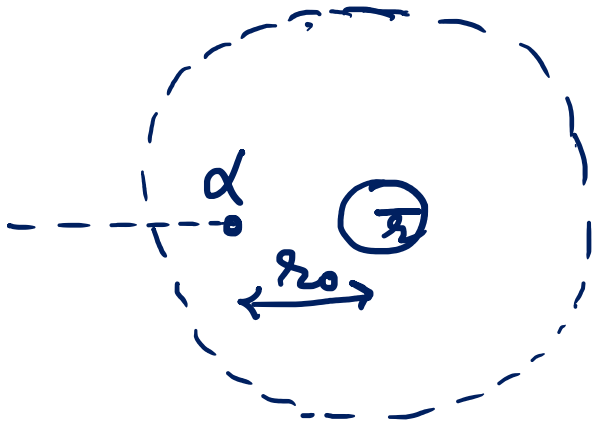


$$U = \frac{kq_1q_2}{r} = \frac{k\,(2e)\,(Ze)}{r}$$

Distance of closest approach (r_0) - give idea about size of nucleus

$r < r_0$

(nucleus) (distance of closest approach)



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* Experiment data suggest that -
  → Most energetic naturally obtained α-particle = 7.7 MeV

Gold foil → $Z = 79$ and $1\,eV = 1.6 \times 10^{-19}\,J$

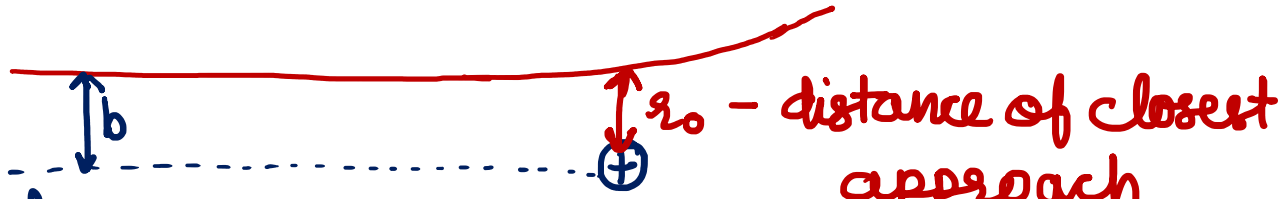
$$r_0 = \frac{2Ze^2}{4\pi\epsilon_0 (KE)}$$

$$r_0 \approx 3 \times 10^{-14}\,m$$
$$\approx 30\,fm$$

in future, we come to know size of gold nucleus $r = 6\,fm$

$[\, r < r_0 \,]$

Impact Parameter (b) - The perpendicular distance of the initial velocity of the α-particle from the centre of the nucleus.



Impact parameter (b)	Scattering angle (θ)
$b \to 0$	$\theta = \pi$
$(b >> r_0)\ b \to \infty$	$\theta = 0°$

Relation between b and θ -

$$b = \frac{kZe^2}{K.E.i}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$\theta = 0° \Rightarrow b \to \infty$

$\theta = \pi \Rightarrow b \to 0$

$k \to$ constant $= 1.77$ MeV

$Z \to$ Atomic number

$K.E.i \to$ initial kinetic energy of α

$\theta \to$ scattering angle

$1\,eV = 1.6 \times 10^{-19}\,J$

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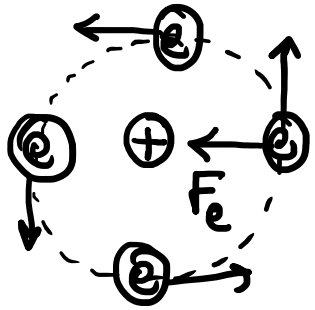
Limitations of Rutherford Model -

i. Stability of Atoms - According to Maxwell (Classical electromagnetism)



⇒ An accelerated charged particle is a source of electromagnetic radiation (energy).

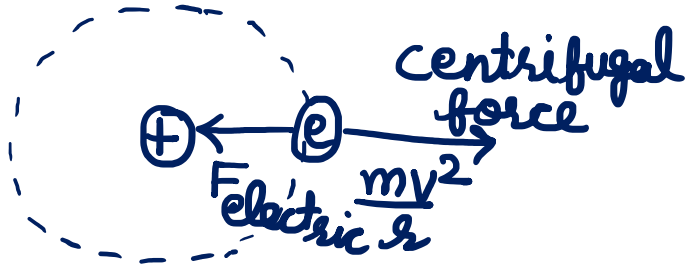
⇒ An electron is accelerated + An electron is charged ⇒ emit radiation (energy)
(direction of V changes)

* electron will loose energy continously (K.E. - decrease ⇒ Velocity - decrease
⇒ Centrifugal Force - decrease ⇒ electric force pull electron into nucleus)

⇒ Calculation shows that it would take 10^{-8} sec for electron to fall into nucleus and atom won't exist.

⇒ This doesn't happen

⇒ Stability of atom cannot be explained.



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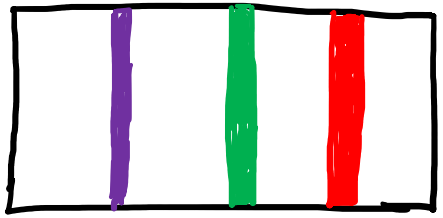
(ii) Discrete Spectrum of Atoms - Atomic Spectrum

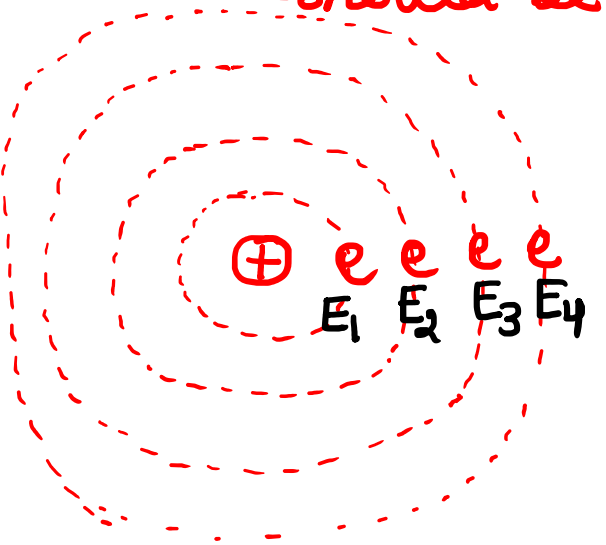
Heated with electric energy → gas emits radiation

Emission Spectrum



Discrete Spectrum ⇓ Containing radiations of certain wavelength only.

(According to Rutherford model ⇒ spectrum of atoms should be continous - which is not correct)



Rutherford → electron revolves in circular orbits.

$$E = E_2 - E_1 \Rightarrow \frac{hc}{\lambda} = E_2 - E_1$$

$$\Rightarrow \lambda = \frac{hc}{E_2 - E_1}$$ (all values → continous spectrum)

↳ which is not correct

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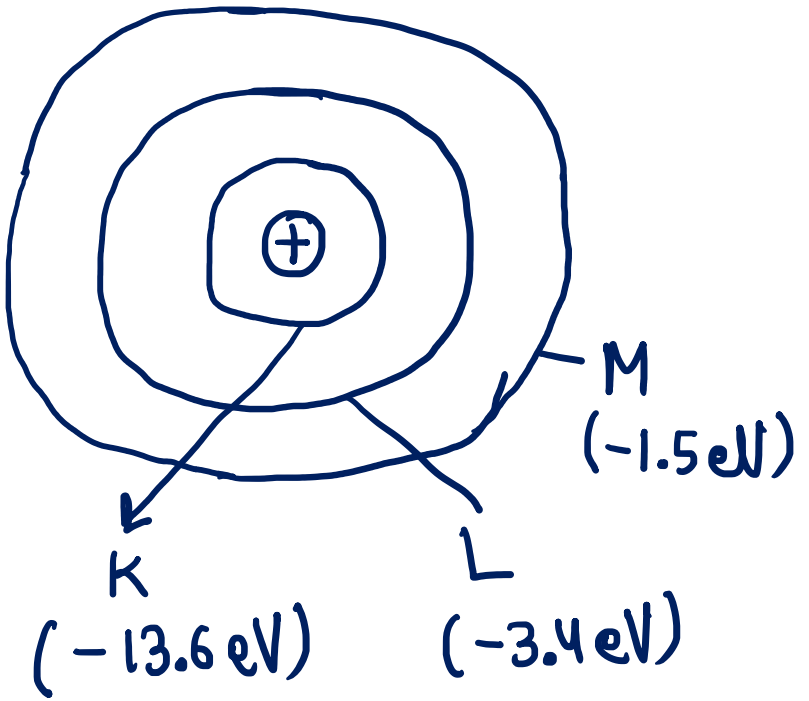
[Bohr - Atomic Model]

Bohr's Posulates -

1. An electron in an atom can revolve only in certain stable orbits with fixed energy. These orbits are called Energy Level or stationary (constant energy) states. These were named as K, L, M, N.... energy levels.

   * An electron will have the energy of its energy level.

2. While revolving in particular energy level, an electron do not radiate any radiations. (Classical electromagnetism and classical mechanics cannot be applied on atomic levels in all aspects.



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3. The energy states are defined as :- (Bohr's Quantisation condition)
electron can revolve only in those orbits for which the orbital angular momentum is integral multiple of $\frac{h}{2\pi}$.

$$L = \frac{h}{2\pi}, \frac{2h}{2\pi}, \frac{3h}{2\pi} \text{ ----}$$

$$\boxed{L = \frac{nh}{2\pi}} \quad n = 1, 2, 3, 4, \text{ - - - -}$$
$$K, L, M, N, \text{ - - -}$$

Orbital angular momentum (L) of e^- in K, L, M, N

$n=1$ (Ground state) $\rightarrow$ K $\longrightarrow$ $L = \frac{h}{2\pi}$

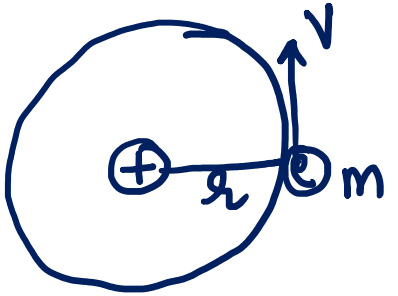
$n=2$ (First excited state) $\rightarrow$ L $\rightarrow$ $\frac{2h}{2\pi}$

$n=3$ (Second excited state) $\rightarrow$ M $\longrightarrow$ $\frac{3h}{2\pi}$

⋮

Bohr Quantisation Condition -

$$\boxed{mvr = \frac{nh}{2\pi}}$$



4. From Rutherford's Model -

An electron moves in a circle, so it need centripetal force. This centripetal force is provided by the electrostatic force between electron and nucleus.

$$F_e = F_c \Rightarrow \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = \frac{mv^2}{r} \Rightarrow \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

#Radius of nth shell -

$$mvr = \frac{nh}{2\pi} \quad \text{— (i)} \qquad \text{and} \qquad \frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} \quad \text{— (ii)}$$

Squaring (i) and divided by (ii)

$$\frac{m^2v^2r^2}{\frac{mv^2}{r}} = \frac{\frac{n^2h^2}{4\pi^2}}{\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}} \Rightarrow mr^2 = \frac{\epsilon_0 n^2 h^2}{\pi Ze^2} r \Rightarrow \boxed{r = \frac{n^2h^2\epsilon_0}{\pi m Ze^2}}$$

or $\boxed{r_{nth} = 0.529\left(\frac{n^2}{Z}\right)\text{Å}}$ or $\boxed{r_{nth} = a_0\left(\frac{n^2}{Z}\right)\text{Å}}$

Velocity of electron in n^{th} Electron -

$$mvr = \frac{nh}{2\pi} \quad \text{and} \quad r = \frac{n^2h^2\epsilon_0}{\pi m Ze^2} \Rightarrow \frac{mvn^2h^2\epsilon_0}{\pi m Ze^2} = \frac{nh}{2\pi}$$

$$\therefore \quad \boxed{V = \frac{Ze^2}{2nh\epsilon_0} = 2.18 \times 10^6 \left(\frac{Z}{n}\right) m/s} \quad \text{or} \quad V = V_0\left(\frac{Z}{n}\right) m/s$$

Q2. Find the ratio of velocity of electron in 2nd excited state of Li^{++} and first excited state of H.

Sol.

$$\frac{V_{2nd\ excited\ Li^{++}}}{V_{1st\ excited\ H}} = \frac{V_0 \times \left(\frac{3}{3}\right)}{V_0 \times \left(\frac{1}{2}\right)} = \frac{1}{\frac{1}{2}} = 2$$

Energy of orbit / electron in n^{th} orbit –

a) $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Ze^2}{2h\epsilon_0 n}\right)^2 = \frac{1}{2}\frac{mZ^2e^4}{4h^2\epsilon_0^2 n^2} \Rightarrow \boxed{K.E. = \frac{mZ^2e^4}{8h^2\epsilon_0^2 n^2}}$

b) $P.E.(U) = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{(-e)(Ze)}{n^2h^2\epsilon_0} \times \pi mZe^2 = -\frac{mZ^2e^4}{4n^2h^2\epsilon_0}$

$\Rightarrow \boxed{U = \frac{-2mZ^2e^4}{8n^2h^2\epsilon_0^2}}$

c) Total energy = $K.E + P.E. = \frac{mZ^2e^4}{8n^2h^2\epsilon_0^2} - \frac{2mZ^2e^4}{8n^2h^2\epsilon_0^2} = -\frac{mZ^2e^4}{8n^2h^2\epsilon_0^2}$

$\Rightarrow \boxed{T.E. = -\frac{mZ^2e^4}{8n^2h^2\epsilon_0^2}}$ or $\boxed{T.E. = \frac{U}{2} = -K.E.}$

$\Rightarrow T.E. = -\left(\frac{me^4}{8h^2\epsilon_0^2}\right) \times \frac{Z^2}{n^2} = -13.6 \times \left(\frac{Z^2}{n^2}\right)$ eV

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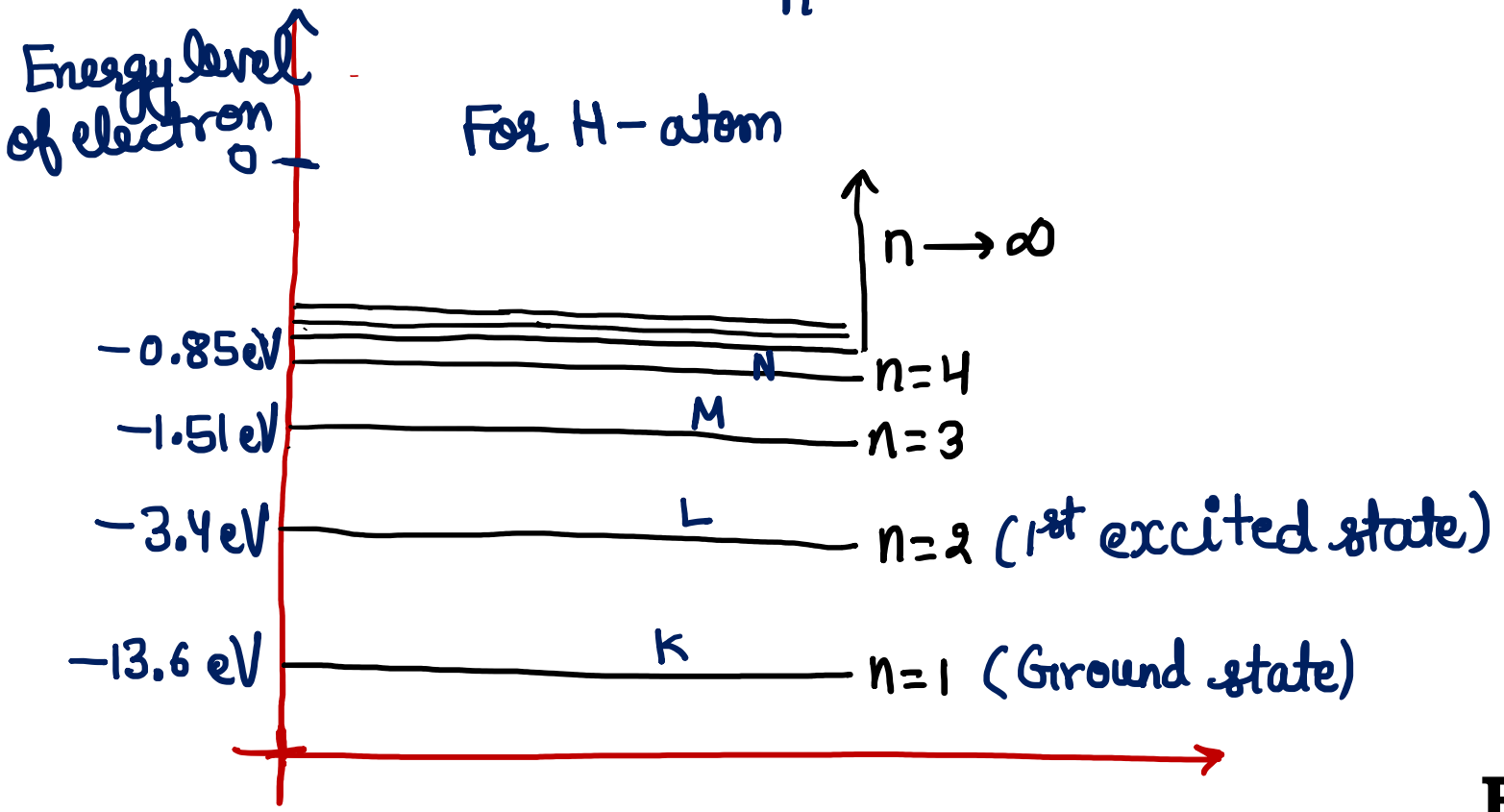
→ Total energy of an electron is negative
→ Electron is in a bounded system.

$$T.E. = -13.6 \times \left(\frac{Z^2}{n^2}\right) \text{ eV}, \quad U = 2(T.E.) = -2 \times 13.6 \times \left(\frac{Z^2}{n^2}\right) \text{ eV}$$

$$K.E. = -T.E. = 13.6 \times \frac{Z^2}{n^2} \text{ eV}$$



Q3. Find the K.E. of an electron in the first excited state of Li^{++} ion.

Sol.

$$E = -13.6 \times \frac{Z^2}{n^2} \text{ eV}$$

$$Li^{++} \begin{cases} Z = 3 \\ n = 2 \end{cases} \Rightarrow E = -13.6 \times \frac{(3)^2}{(2)^2} = -3.4 \times 9 = -30.6 \text{ eV}$$

$$E = \frac{U}{2} = -\text{K.E.} \Rightarrow \text{K.E.} = -E = 30.6 \text{ eV}$$

Q4. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization condition are applied, radii of possible orbitals and energy levels vary with quantum number n as –

a) $r_n \propto n$, $E_n \propto n$

b) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$

c) $r_n \propto \sqrt{n}$, $E_n \propto n$

d) $r_n \propto \sqrt{n}$, $E_n \propto \frac{1}{n}$

Sol. $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$.

Q5. A He^+ ion is in its first excited state. Its ionization energy is-

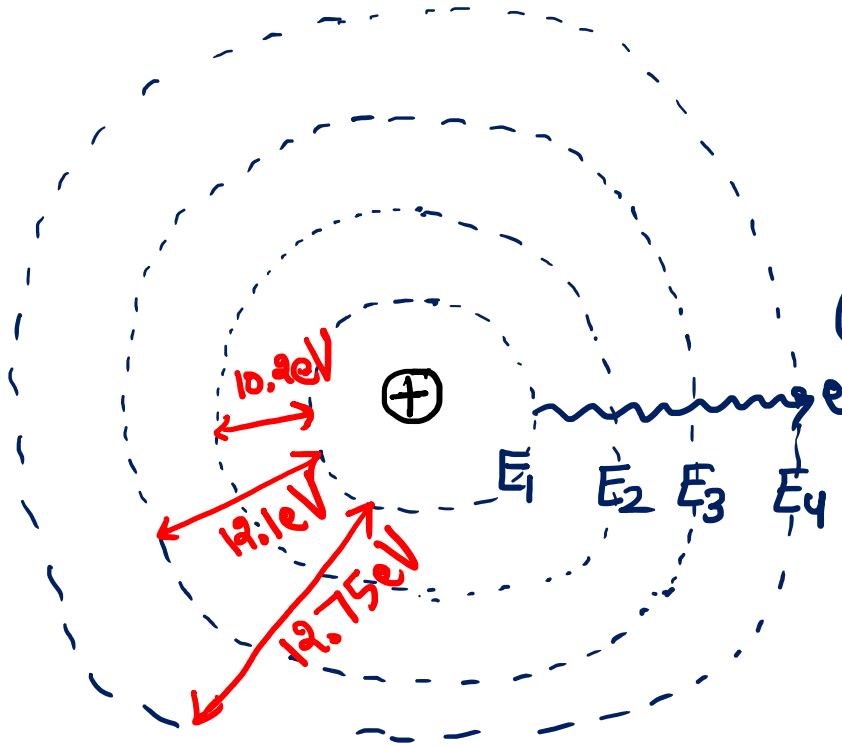
a) 54.40 eV b) 13.60 eV c) 48.36 eV d) 6.04 eV

Sol -

$$E = -13.6 \times \left(\frac{Z^2}{n^2}\right) eV = -13.6 \times \frac{2^2}{2^2} = -13.6 \text{ eV}$$

$\therefore$ Ionisation energy $= -E = -(-13.6) = 13.6$ eV

For Hydrogen atom -



$E_1 = -13.6$ eV
$E_2 = -3.4$ eV
$E_3 = -1.5$ eV
$E_4 = -0.85$ eV

$(E_1 < E_2 < E_3 < E_4)$ energy increase

An electron while orbiting in a lower energy level may absorbs a specific photon (energy) and jumps (excites) to a higher energy level. The energy of such a photon is equal to energy difference of two orbits

$$\Delta E = E_{n_2} - E_{n_1} \quad \text{or} \quad \frac{hc}{\lambda} = E_{n_2} - E_{n_1}$$

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An electron in a higher energy level may release a specific photon (energy) and jumps (dexcite) (makes transition) to a lower energy level. The energy of emitted photon is equal to the energy difference of two orbits.

$$\Delta E = E_{n_2} - E_{n_1} \Rightarrow \frac{hc}{\lambda} = E_{n_2} - E_{n_1}$$

Q6. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state in the excited state in terms of Bohr radius a_0 will be – (Take $hc = 12500\ eV-Å$)

a) $4a_0$ b) $9a_0$ c) $16a_0$ d) $25a_0$

Sol $r_{nth} = a_0 \times \frac{n^2}{Z}\ Å = a_0 n^2$, $\Delta E = \frac{hc}{\lambda} = \frac{12500\ eV\ Å}{980\ Å} = \frac{1250}{98} = 12.7 eV$

$$E_{n_2} = 12.7 - 13.6 = -0.9 \simeq -0.85 \Rightarrow n = 4$$

$$\Rightarrow r_{nth} = a_0 \times n^2 = 16a_0$$

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Q7. Find the wavelength of radiation emitted when electron in Hydrogen atom jump from $n=3$ to $n=2$.

Sol. $\Delta E = E_3 - E_2 = -13.6 \times \frac{Z^2}{n^2} - \left(-13.6 \times \frac{Z^2}{n^2}\right) = -13.6 \times \frac{1}{3^2} + 13.6 \times \frac{1}{2^2} = 1.9\ eV$

$\Delta E = \frac{hc}{\lambda} = \frac{1240}{\lambda\ (\text{in nm})} \Rightarrow 1.9 = \frac{1240}{\lambda} \Rightarrow \lambda = \frac{1240}{1.9} = 650\ nm$

Drawbacks in Bohr Model -

(i) Model was applicable only for H-atom and H-like atoms. Single electron species- He^+, Li^{++}, Be^{+++} or isotopes of hydrogen ($_1H^1$, $_1H^2$, $_1H^3$)

(ii) It could not explain spectra of multielectron (inconsistent with De-Broglie hypothesis).

(iii) Consider only particle nature of electron, wave nature of electron was not taken into consideration (inconsistent with De Broglie hypothesis).

(iv) It violates Heisenberg's uncertainity principle and simultaneously defines

position (value of r_n) and momentum (value of v) of electron correctly.

[Heisenberg's Uncertainity Principle - it is impossible to define correctly and simultaneously the position and momentum of a microscopic moving particle.

* Hydrogen Atom -

position → $r_1 = 0.529\ \text{Å}$

momentum → $mv_1 = 9.1 \times 10^{-31}\ \text{kg} \times 2.18 \times 10^{6}\ \text{m/s}$

[Atomic Spectrum]

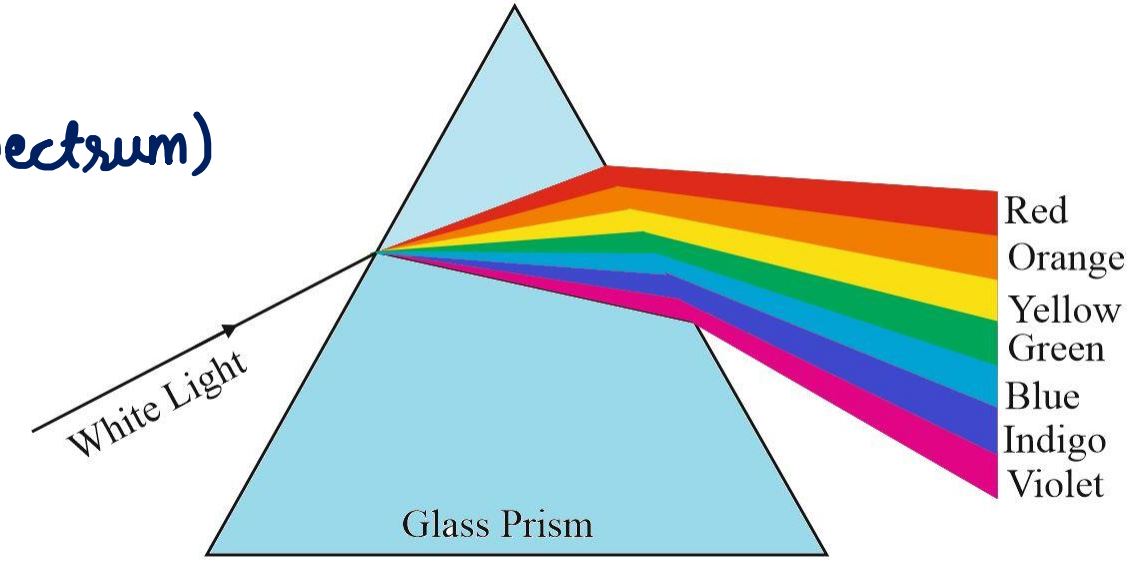
Dispersion and Spectrum -

Dispersion - VIBGYOR (Continous Spectrum)

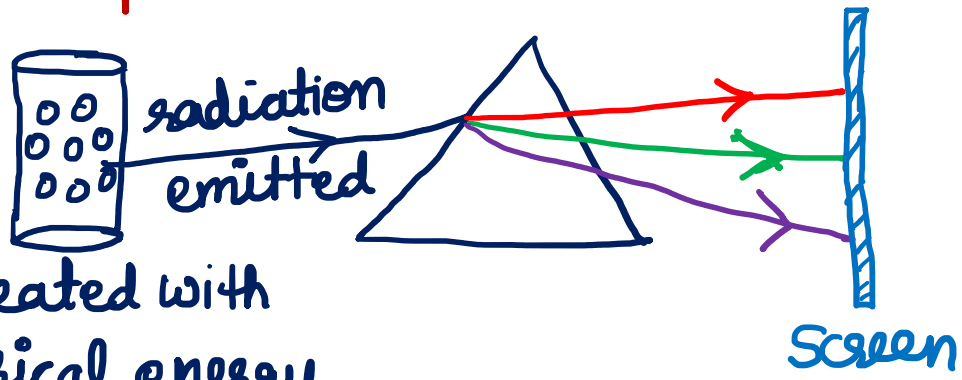
There are two types of Atomic Spectrum -

(i) Emission Spectrum

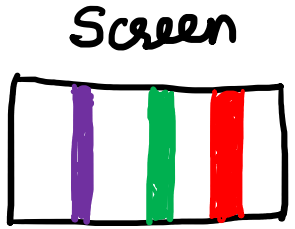
(ii) Absorption Spectrum



Emission Spectrum -



gas heated with electrical energy



certain specific wavelength (λ) are obtained $\Rightarrow$ Discrete spectrum $\Rightarrow$ Line spectrum

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→ Explanation for Line spectrum or discrete spectrum –

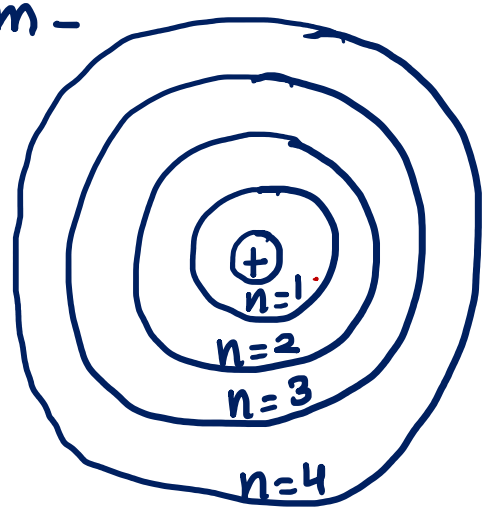
1. According to Bohr Atomic Model – there are certain energy states in which electron can be found. ($E_K < E_L < E_M < E_N$)

* Energy of emitted radiation (ΔE) for hydrogen atom –

$$\Delta E = E_{n_2} - E_{n_1} = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_{n_2} - E_{n_1}}$$

$n_1 \rightarrow -13.6$ eV
$n_2 \rightarrow -3.4$ eV
$n_3 \rightarrow -1.5$ eV
$n_4 \rightarrow -0.85$ eV

$n_2 - n_1 = 10.2$ eV
$n_3 - n_1 = 12.1$ eV
$n_4 - n_1 = 12.75$ eV



$$\lambda = \frac{hc}{E_{n_2} - E_{n_1}}$$

Higher — E_{n_2}, Lower — E_{n_1}

→ E_{n_2} and E_{n_1} can take only some fixed values

→ $E_{n_2} - E_{n_1}$ can have certain fixed values.

→ λ will have certain fixed values.

Q8. Calculate the wavelength (in Å) of a photon emitted when an electron in a hydrogen atom falls from n=3 to n=1.

a) 600 Å b) 1024 Å c) 1414 Å d) 3600 Å

Sol. $\Delta E = E_{n_2} - E_{n_1} \Rightarrow \frac{hc}{\lambda} = E_3 - E_1 \Rightarrow \frac{hc}{\lambda} = -13.6\left(\frac{1^2}{3^2}\right) - \left(-13.6 \times \frac{(1^2)}{(1^2)}\right)$

$\Rightarrow \frac{hc}{\lambda} = 12.09\ eV$ and $\Delta E\ (in\ eV) = \frac{1240}{\lambda\ (in\ nm)}$

$\therefore 12.09 = \frac{1240}{\lambda} \Rightarrow \lambda = 103.3\ nm = 1033\ nm\ (approx)$

Formula to calculate wavelength (λ) of emitted photon -

$\Delta E = E_{n_2} - E_{n_1}$ and $\frac{hc}{\lambda} = -13.6\left(\frac{Z^2}{n_2^2}\right) - \left(-13.6\left(\frac{Z^2}{n_1^2}\right)\right) = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow \frac{1}{\lambda} = \frac{13.6}{hc}Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$

where R - Rydberg Constant, $R = \frac{13.6}{hc}eV = 1.09 \times 10^7\ m^{-1}$

and $\frac{1}{R} \approx 911\ Å$

By- Sonu Kumar

Q9. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths $\frac{\lambda_1}{\lambda_2}$ of the photons emitted in this process is

a) $\frac{20}{7}$ b) $\frac{27}{5}$ c) $\frac{7}{5}$ d) $\frac{9}{7}$

Sol. $\lambda_1 \rightarrow n=4$ to $n=3$

$$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = R\left(\frac{7}{9\times16}\right)$$

$\lambda_2 \rightarrow n=3$ to $n=2$

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{5}{4\times9}\right)$$

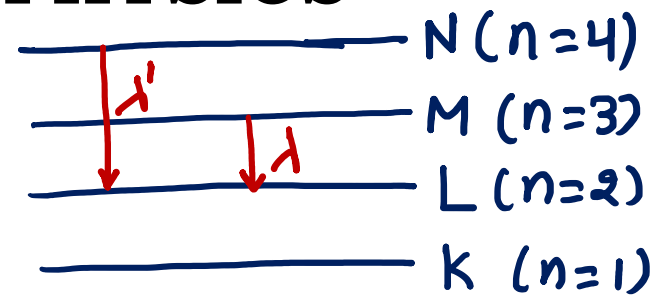
3rd excited — n = 4
2nd excited — n = 3
1st excited — n = 2
Ground State — n = 1

$$\frac{\frac{1}{\lambda_2}}{\frac{1}{\lambda_1}} = \frac{R\left(\frac{5}{4\times9}\right)}{R\left(\frac{7}{9\times16}\right)} = \frac{5}{4}\times\frac{16}{7} = \frac{20}{7}$$

Q10. In a hydrogen like atom, when an electron jumps from the M-Shell to the L-Shell, the wavelength of emitted radiation is λ. If an electron jumps from N-Shell to L-Shell, the wavelength of emitted radiation will be -

a) $\frac{27}{20}\lambda$ b) $\frac{25}{16}\lambda$ c) $\frac{20}{27}\lambda$ d) $\frac{16}{25}\lambda$

Bal.



λ (n=3 to n=2)

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{4} - \frac{1}{9}\right) = RZ^2\left(\frac{5}{4\times 9}\right)$$

λ' (n=4 to n=2)

$$\frac{1}{\lambda'} = RZ^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = RZ^2\left(\frac{3}{16}\right)$$

$$\therefore \quad \frac{\frac{1}{\lambda}}{\frac{1}{\lambda'}} = \frac{RZ^2\left(\frac{5}{4\times 9}\right)}{RZ^2\left(\frac{3}{16}\right)} = \frac{5}{4\times 9} \times \frac{16}{3} = \frac{20}{27} \Rightarrow \lambda' = \frac{20}{27}\lambda$$

Q11. Hydrogen $({}_1H^1)$, deuterium $({}_1H^2)$, singly ionised helium $({}_2He^4)^+$ and doubly ionised Lithium $({}_3Li^8)^{++}$ all have one electron around the nucleus. Consider an electron transition from n=2 to n=1. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively for four elements, the approximately which one of the following is correct?

a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

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Sol. λ (n=2 to n=1)

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(Z^2) = kZ^2$ for each element $\left(k = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)\right)$

$\Rightarrow \lambda = \frac{1}{kZ^2}$

${}_1H^1$, $\lambda_1 = \frac{1}{k(1)^2} = \frac{1}{k}$

${}_1H^2$, $\lambda_2 = \frac{1}{k(1)^2} = \frac{1}{k}$

${}_2He^{4+}$, $\lambda_3 = \frac{1}{k(2)^2} = \frac{1}{4k}$

${}_3Li^{8+}$, $\lambda_4 = \frac{1}{k(3)^2} = \frac{1}{9k}$

$k = \frac{1}{\lambda_1} = \frac{1}{\lambda_2} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4} \Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

[Spectral Series]

1. Lyman Series –

$n_1 = 1$, $n_2 = 2, 3, 4 \ldots \infty$

all wavelength (λ) corresponding to this transition.

For hydrogen atom ($Z = 1$),

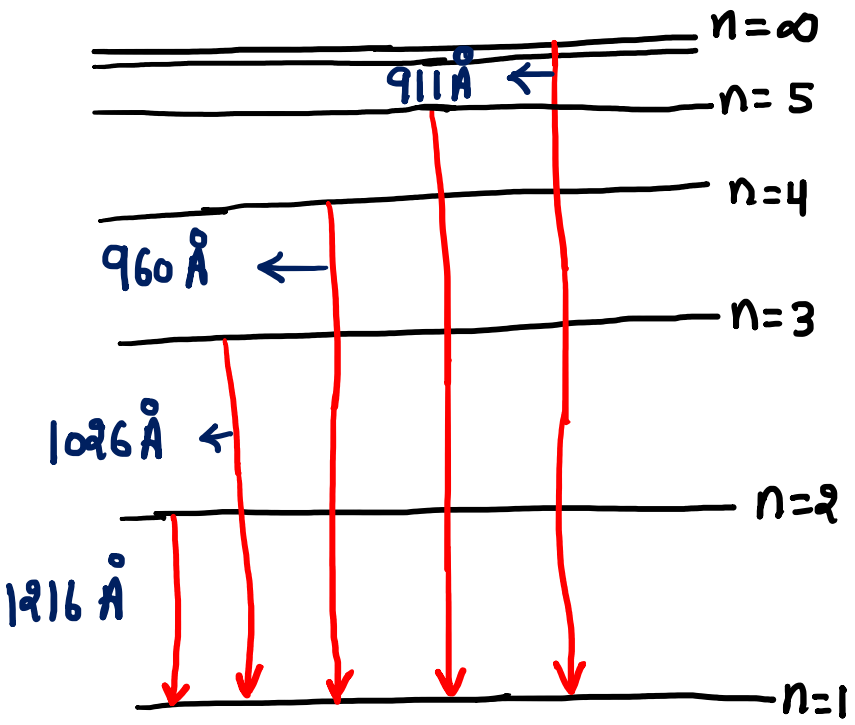
a) $\frac{1}{\lambda} = R(1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R \Rightarrow \lambda = \frac{4}{3R}$

$\therefore \lambda = \frac{4}{3} \times \frac{1}{R} = \frac{4}{3} \times 912\ \text{Å} = 1216\ \text{Å}$ (largest wavelength for Lyman series)

b) $\frac{1}{\lambda} = R(1)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) \Rightarrow \lambda = 1026\ \text{Å}$

c) $\frac{1}{\lambda} = R(1)^2\left(\frac{1}{1^2} - \frac{1}{4^2}\right) \Rightarrow \lambda = 960\ \text{Å}$

d) $\frac{1}{\lambda} = R(1)^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) \Rightarrow \lambda = 911\ \text{Å}$ (smallest wavelength for Lyman series)



* largest $\lambda \Rightarrow$ smallest transition
* smallest $\lambda \Rightarrow$ largest transition

Discrete spectrum

By- Sonu Kumar

2. Balmer Series –

$n_1 = 2$, $n_2 = 3, 4, 5 \ldots \infty$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

a) Longest wavelength (λ) of Balmer series $\Rightarrow$ smallest transition (1st line) $\Rightarrow n_2 = 3, n_1 = 2$

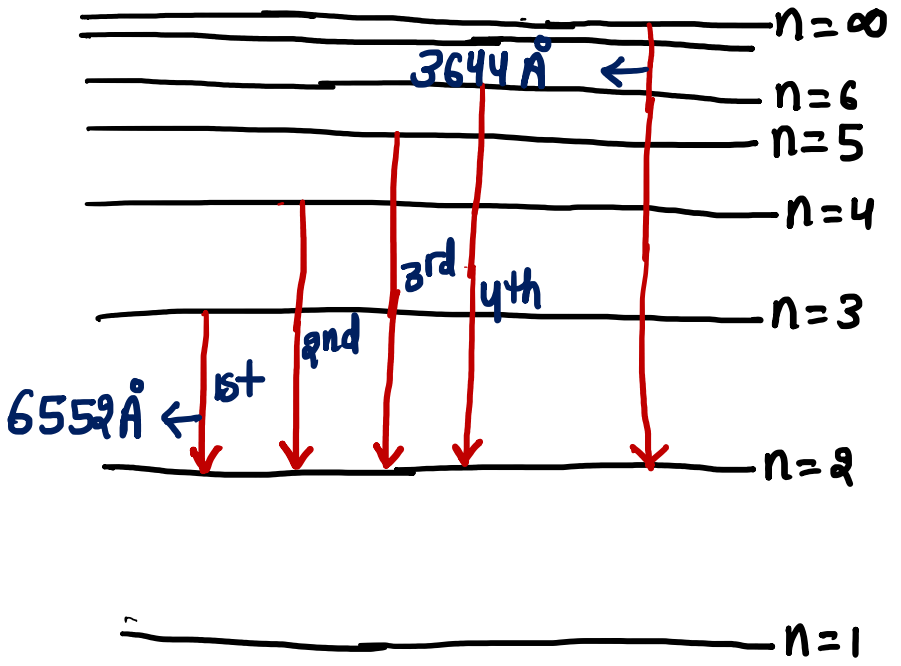
For H-atom,

$$\frac{1}{\lambda} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda = 6552\,\text{Å}$$

b) Smallest wavelength of Balmer series $\Rightarrow$ longest transition ($n_2 = \infty, n_1 = 2$)

For H-atom,

$$\frac{1}{\lambda} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) \Rightarrow \lambda = 3644\,\text{Å}$$



Q 12. Taking the wavelength of first Balmer line in hydrogen spectrum ($n=3$ to $n=2$) as 660 nm, the wavelength of the 2nd Balmer line ($n=4$ to $n=2$) will be

a) 889.2 nm b) 338.9 nm c) 642.7 nm d) 488.9 nm

Sol. First Balmer Line ($Z=1$) ($n=3$ to $n=2$)

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = R\left(\frac{1}{4}-\frac{1}{9}\right) = R\left(\frac{5}{36}\right) \Rightarrow \lambda_1 = 660\,nm$$

Second Balmer Line ($Z=1$) ($n=4$ to $n=2$)

$$\frac{1}{\lambda_2} = R(1)^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = R\left(\frac{1}{4}-\frac{1}{16}\right) \Rightarrow \frac{1}{\lambda_2} = R\left(\frac{3}{16}\right)$$

$$\therefore \frac{\frac{1}{\lambda_1}}{\frac{1}{\lambda_2}} = \frac{R\left(\frac{5}{36}\right)}{R\left(\frac{3}{16}\right)} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{5}{36}\times\frac{16}{3} = \frac{20}{27} \Rightarrow \lambda_2 = \frac{20}{27}\lambda_1$$

$$\therefore \lambda_2 = 488\,nm$$

Q13. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is –

a) 1215 Å b) 1640 Å c) 2430 Å d) 4687 Å

Sol. First Spectral line Balmer (H-atom) → $Z=1$ ($n=3$ to $n=2$)

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = R\left(\frac{5}{36}\right) \Rightarrow \lambda_1 = \frac{36}{5R}$$

Second spectral line Balmer (He-atom) → $Z=2$ ($n=4$ to $n=2$)

$$\frac{1}{\lambda_2} = R(2)^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = R\left(\frac{3}{4}\right) \Rightarrow \lambda_2 = \frac{4}{3R}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{5R}{36}\times\frac{4}{3R} = \frac{5}{27} \Rightarrow \lambda_2 = \frac{5}{27}\lambda_1 = \frac{5}{27}\times 6561 = 1215\text{Å}$$

→ Longest wavelength of Pfund series -

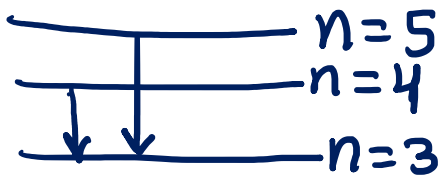
longest λ → shortest transition

$(6 \rightarrow 5)$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{5^2} - \frac{1}{6^2}\right)$$

Series		Region
Lyman Series	$n=1$	Ultraviolet region
Balmer Series	$n=2$	Visible region
Paschen Series	$n=3$	Low infrared region
Bracket Series	$n=4$	infrared region
Pfund Series	$n=5$	infrared region
Humphrey Series	$n=6$	High infrared region

→ Second line of Paschen Series-



→ Find the shortest wavelength of visible region ?

Visible region → Balmer series ($n_1=2$ and n_2 = from ∞ to 2)

Q.14. If the series limit frequency of the Lyman Series is ν_L, then the series limit frequency of the Pfund series is -

a) $\frac{\nu_L}{25}$ b) $16\nu_L$ c) $\frac{\nu_L}{16}$ d) $\frac{\nu_L}{20}$

Sol. Series limit (max.) frequency of Lyman series –

$\nu_{max} \Rightarrow \lambda_{min} \Rightarrow$ longest transition ($n_1 = 1$ and $n_2 = \infty$)

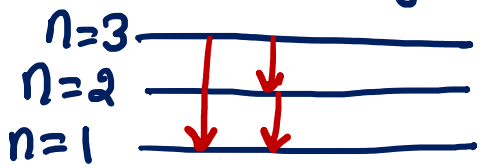
$$\Rightarrow \frac{1}{\lambda_L} = RZ^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = RZ^2$$

Series limit frequency of Pfund series – ($n_1 = 5$ and $n_2 = \infty$)

$$\Rightarrow \frac{1}{\lambda_P} = RZ^2\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = \frac{RZ^2}{25}$$

$$\therefore \frac{\frac{1}{\lambda_L}}{\frac{1}{\lambda_P}} = \frac{RZ^2}{\frac{RZ^2}{25}} \Rightarrow \frac{\lambda_P}{\lambda_L} = 25 \Rightarrow \frac{\nu_L}{\nu_P} = 25$$

Number of transitions / Number of spectral lines / Number of unique wavelengths –
(How many types of wavelength appear)



$$\text{Number of spectral lines} = \frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$$

Q15. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets de-excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ? [Take $h = 6.63 \times 10^{-34}$ Js]

a) 9.4 nm b) 12.3 nm c) 10.8 nm d) 11.4 nm

Sol. Number of spectral line $= \dfrac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$

$$6 = \frac{(n-1)(n-1+1)}{2} \Rightarrow 12 = (n-1)n \Rightarrow n = 4$$

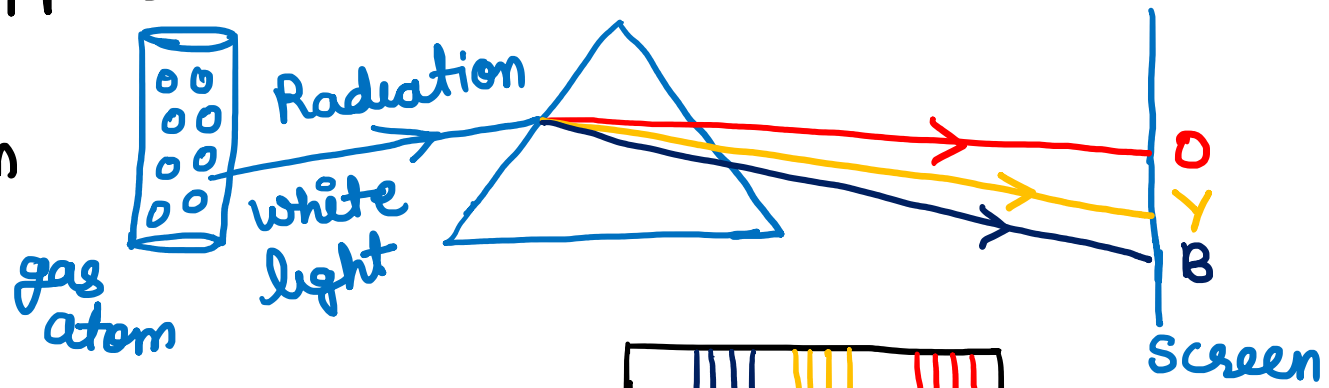
For Li^{++}, $(n = 4 \text{ to } n = 1)$, $Z = 3$

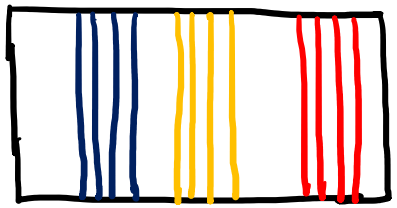
$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = R \times 9 \times \frac{15}{16} \Rightarrow \lambda = \frac{16}{15} \times \frac{1}{9} \times \frac{1}{R} = \frac{16}{15} \times \frac{1}{9} \times 910\ \text{Å}$$

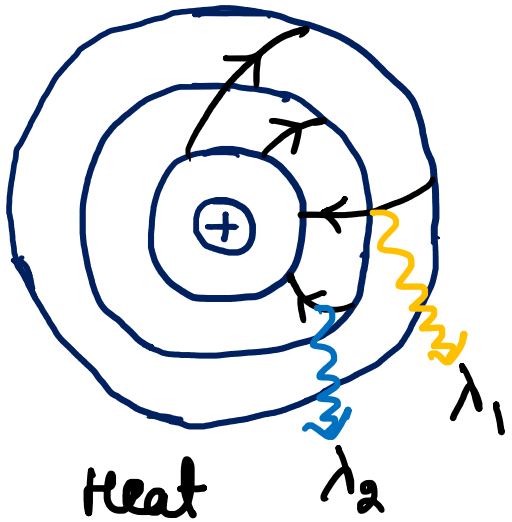
$\therefore \lambda = 107.7\ \text{Å}$ or $10.77\ \text{nm}$

[Absorption Spectrum]

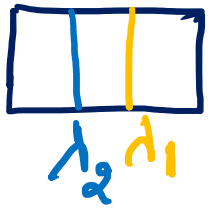
→ All the radiations (λ) which appear in emission spectrum of a particular gas, are missing in absorption spectrum of that gas.

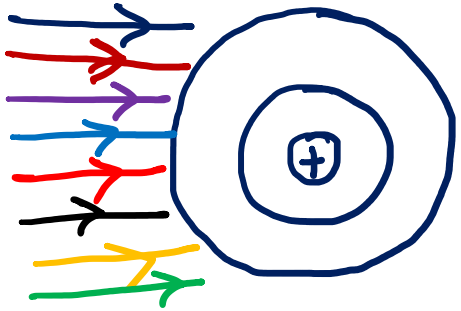


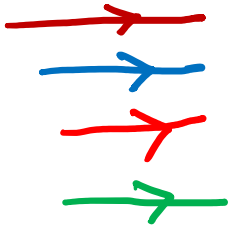




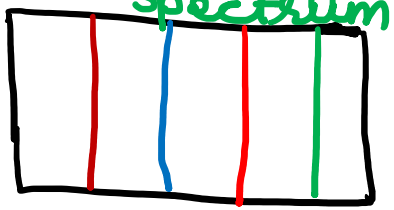
emission Spectrum







Absorption spectrum



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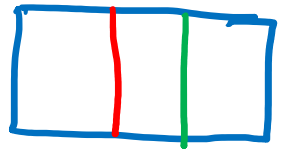
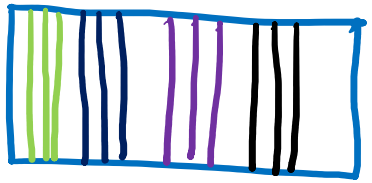
→ In the emission spectrum, only certain radiations (with certain λ) are seen (discrete lines are seen).



In apsorption spectrum, those certain radiations (with certain λ) (which appears in emission) are missing. (discrete lines are missing)